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THE EQUIMARGINAL PRINCIPLE MEANING, LIMITS, AND GENERALIZATION

by
MAURICE ALLAIS *

1. *The equimarginal principle* was discovered first by Gossen (1854), and rediscovered, broadened and introduced independently into economics by Jevons (1871), Menger (1871) and Walras (1874-1877). In the following years numerous new developments of the principle have been presented by their immediate successors, especially by Edgeworth (1881), Irving Fisher (1892) and Vilfredo Pareto (1896-1911).

This principle corresponds to *the outcome of the dynamic process of the economy induced by differences in marginal equivalences*. According to Irving Fisher, with whose judgement I agree fully, "*No idea has been more fruitful in the history of economic science*". Its applications and generalizations dominate all economic analysis in real terms.

The history of the progressive development of the dynamic theory of marginal equivalences and its generalizations goes back to the 18th century, and its guiding principles are present more or less clearly in such authors as Boisguilbert (1695 and 1705), Turgot (1766) and Condillac (1776). Particularly striking illustrations are Ricardo's theory of comparative costs (1817) and Dupuit's theory of economic losses (1844-1854).

Despite some too frequent belief to the contrary, there has never been a "*marginal revolution*". As of the XVIIIth century, all the materials for the building of the equimarginal theory were present in embryo and ready for implementation, but there was not the slightest inkling of the scope it was to have. It is at the same time the meditation on the writings of earlier authors and the use of differential calculus which permitted the explosion of the marginal thought.

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From Walras on, the literature becomes *progressively – and unduly –* concentrated on equilibrium analysis which, however interesting it could be, is *less* so than the analysis of the processes by which the economy tends at any time towards situations of equilibrium which in fact are never reached.

The theory of marginal equivalences *assumes* that quantities are continuous and functions differentiable. These hypotheses were and are *very useful* in allowing very essential aspects to be brought to light. But they are also *over restrictive* and too often unrealistic in that very many economic magnitudes do not vary continuously, and that indivisibilities often play a major role.

The general theory of surplus which I developed from 1943 to 1967, and thoroughly analysed from 1967 to 1985, appears essentially *as a double generalization of marginal theory.* On the one hand it considers *the most general* modifications of the economy, given its structural conditions, instead of the bilateral relationships among pairs of economic operators. On the other hand, even more important, it extends marginal analysis *to the case of discrete changes,* the hypotheses of continuity and differentiability being only considered as corresponding to very special cases. This theory attaches much *more importance to the processes which enable the economy to approach situations of equilibrium and maximum efficiency than to these situations per se,* whose intrinsic interest for understanding how the economic world actually functions or as a basis for formulating economic policy is rather limited.

The purpose of the following is to analyze *the equimarginal principle as regards its meaning, limits and generalizations.* To this end, *Part I* first gives a short statement of the basic economic concepts and propositions *in the most general case* in which the hypotheses of continuity and differentiability, which are implied by the equimarginal principle, cannot be admitted. Taking the hypotheses of continuity and differentiability, *Part II* then presents an analysis of the equimarginal principle considered both as the outcome of the dynamic process stemming from the differences in marginal equivalences and as a condition of stable equilibrium and maximum efficiency. *Part III* specifies the actual meaning and the limits of application of the equimarginal principle. *Part IV* gives an illustration of marginal analysis in the case of production units with increasing returns. *Part V* presents some brief comments on some mythologies of the contemporary literature and the immense regression of the vast majority of its theories by comparison with the classical marginal approach. *Part VI* presents a brief historical overview and specifies the conditions for a realistic analysis of the dynamic process of the economy in real terms for future prospects of economic thought.

The basic concepts and propositions are presented in the framework of a very *general model which is free of any restrictive condition, namely the model of a markets economy* (in contradistinction to a "*market economy*") which I proposed in 1967, and a theory, *the general theory of surpluses*, which I devised from 1946 to 1967, generalizing the classical marginal theory (Part I). In the case of continuity and differentiability, this model and this theory illustrate significantly the implications of the *equimarginal principle*, together with its limits, difficulties of application and generalizations (Part II, III, and IV).

Part V places the approach followed *in relation to the guiding principles of the contemporary theories descending from Walras*, and which in fact constitute an impediment to any genuine progress in analysis of the economy in real terms: the adoption of the market economy model, the principle that a common price system applicable to all operators prevails at each instant, the assumption of general convexity, and the exaltation of mathematical formalism of theory of sets to the detriment of the conformity with actual facts. *By departing from the great tradition of marginal theory* and by adopting an unrealistic model and unrealistic assumptions, the contemporary theories, purely mathematical, have doomed themselves to sterility as regards the understanding of reality. To illustrate: not only are they unable to analyse either situations of increasing returns or the essential phenomenon of indivisibilities, but they are unable to integrate the results of marginal analysis even when they are valid. Only by comparing the bases of the contemporary theories and those of the approach of surpluses and the economy of markets will the reader achieve a full grasp of what is at issue.

Although the purpose of the following analysis is to present *a very brief overview* of the subject, it will nevertheless deal explicitly and suitably with the aspects which generally give rise to inadequate analyses, and even indeed to errors of interpretation. An attempt will be made in particular to demistify some of the mythologies of the contemporary literature, which, based on a flood of absurd hypotheses and theorems which do not correspond to any real problem, lead economic thought into sterile and blind paths. In so doing, it is inspired by Jevons' statement: "*It is impossible that one who has any regard for truth can long avoid protesting against doctrines which seem to him to be erroneous*".

I. – THE GENERAL THEORY OF SURPLUSES AND THE ECONOMY OF MARKETS FUNDAMENTAL CONCEPTS AND THEOREMS

Marginal analysis is actually only a particular case of a more general theory, the theory of surpluses and the economy of markets, which, if considered first, facilitates the discussion of the meaning, limits and generalizations of the *equimarginal principle.*

11. *Hypotheses*

To simplify the exposition, it is assumed that a same good (U) enters all preference and production functions, and that its quantity can vary continuously. *Except for the hypothesis of continuity for this good* (U), *the discussion in this first part is free of any restrictive hypothesis of continuity, derivability or convexity for the goods* (V) ... (W) *considered, and the preference indices and production functions.*

For an exposition of the following theory in the case where no good plays a particular role, see Allais (1985, section II, p. 139-141).

12. *Structural Conditions*

12.1 *Ordinal Preference Indices.* – The needs of every unit of consumption, *individual or collective*, can be entirely defined by considering a preference index

$$I_i = f_i\,(U_i,\ V_i,\ \ldots,\ W_i) \tag{1.1}$$

this being its consumption function, increasing as it passes from a given situation to one it finds preferable. Every quantity V_i is counted *positively* if it refers to a consumption, *negatively* if it refers to a service supplied.

12.2 *Production Functions.* – The set of feasible techniques for a unit of production j can be represented by a condition of the form

$$f_j\,(U_j,\ V_j,\ \ldots,\ W_j) \geq 0 \tag{1.2}$$

where every quantity V_j is considered *as representing a consumption or an output depending on whether it is positive or negative.*

The extreme points corresponding to the boundary between possible and impossible situations represent *states of maximum efficiency for the production unit considered.* They may be represented by the condition

$$f_j\,(U_j,\ V_j,\ \ldots,\ W_j) = 0 \tag{1.3}$$

The function f_j may be called the production function. It is defined up to any transformation which leaves its sign unchanged.

From a technical point of view, maximum efficiency implies quite specific conditions. If *for instance*, one considers a production technique $A = A(X, Y, ..., Z)$ and if n production units are technically preferable to a single one, we should have (Allais, 1943, p. 187-188; 1981, p. 319-322)

$$\sum_j A(X_j, Y_j, ..., Z_j) > A\left[\sum_j X_j, \sum_j Y_j, ..., \sum_j Z_j\right] \quad (1.4)$$

In the opposite case we have

$$A\left[\sum_j X_j, \sum_j Y_j, ..., \sum_j Z_j\right] > \sum_j A\left[X_j, Y_j, ..., Z_j\right] \quad (1.4^*)$$

An industry is referred to as *differentiated* if the use of distinct production units is *technically* more advantageous than the concentration of all production operations into a single production unit. It is called *non-differentiated* in the opposite case. Conditions (1.4) and (1.4*) are two particular illustrations of differentiation and non-differentiation (Allais, 1943, p. 637).

From inequality (1.4) it is possible to show that the whole production function of a differentiated industry is asymptotically homogeneous. In this case ($n \gg 1$) there is quasi-homogeneity (Allais, 1943, p. 201-206; and 1974 B).

13. *Distributable Surplus Corresponding to a Given Modification of the Economy*

The distributable surplus σ_u relating to a good (U) and to a realizable modification of the economy which leaves all preference indices unchanged is defined as the quantity of this good which can be released following this shift (Allais, 1943, p. 610-616).

The surplus considered here essentially *differs* from the concept of *consumer surplus* as normally considered in the literature (Samuelson, 1947, p. 195-202; Coase, 1968; Blaug, 1985, p. 355-370; Allais, 1981, p. 297-298, and 1985, n. 12-13).

13.1 *Definition of distributable surplus.* – Let us consider an initial state ($\mathcal{E}_1$) characterized by consumption values U_i, V_i, ..., W_i and U_j, V_j, ...,

W_j (positive or negative) of the different units of consumption and production. We have

$$\sum_i U_i + \sum_j U_j = U_o;\ \sum_i V_i + \sum_j V_j = V_o;\ ...;\ \sum_i W_i + \sum_j W_j = W_o \quad (1.5)$$

where U_o, V_o, ..., W_o designate available resources.

Let $(\delta\mathcal{E}_1)$ be a feasible modification of $(\mathcal{E}_1)$ characterized by finite variations δU_i, δV_i, ..., δW_i, δU_j, δV_j, ..., δW_j, and let

$$(\mathcal{E}_2) = (\mathcal{E}_1) + \delta(\mathcal{E}_1) \quad (1.6)$$

represent the new state.

According to (1.5) we naturally have

$$\sum_i \delta V_i + \sum_j \delta V_j = 0 \quad (1.7)$$

for every good (U), (V), ..., (W).

We also have for every unit of production j

$$f_j(U_j + \delta U_j,\ V_j + \delta V_j,\ ...,\ W_j + \delta W_j) = 0 \quad (1.8)$$

The preference indices become

$$I_i + \delta I_i = f_i(U_i + \delta U_i,\ V_i + \delta V_i,\ ...,\ W_i + \delta W_i) \quad (1.9)$$

The δI_i can be positive, zero, or negative.

Let us now define a third state $(\mathcal{E}_3)$ by the condition that by the modification $-\delta\sigma_{ui}$ of just the quantities $U_i + \delta U_i$ *all the preference indices return to their initial values.*

We then have the conditions

$$f_i(U_i + \delta U_i - \delta\sigma_{ui},\ V_i + \delta V_i,\ ...,\ W_i + \delta W_i) = f_i(U_i,\ V_i,\ ...,\ W_i) \quad (1.10)$$

The state $(\mathcal{E}_3)$ can be termed "*isohedonous*" with the state $(\mathcal{E}_1)$. In passing from $(\mathcal{E}_1)$ to $(\mathcal{E}_3)$ the quantity

$$\delta\sigma_u = \sum_i \delta\sigma_{ui} \quad (1.11)$$

of the good (U) is *released*, as all the units of consumption find themselves again in situations which they consider equivalent, since their preference indices return to the same values (Allais, 1943, p. 637-638).

The surplus $\delta\sigma_u$ has been released during the passage from $(\mathcal{E}_1)$ to $(\mathcal{E}_3)$. It may then be considered that in the situation $(\mathcal{E}_1)$ this surplus was both *realizable* and *distributable*. It may further be considered that in passing from ($\mathcal{E}_1$) to ($\mathcal{E}_2$), it has in effect been *distributed*.

The distributable surplus thus defined covers the whole economy, but this definition can be used for any group of operators. It is necessary only to consider the functions f_i and f_j and the resources relating to this group in the preceding relations.

13.2 *System of specific prices corresponding to the realization of a distributable surplus.* – Any exchange system, with the corresponding production operations it implies, is deemed "*advantageous*" when a distributable surplus is achieved and distributed, so that the preference index of any consumption unit concerned increases.

In fact if an exchange and production system is advantageous, there is at least one system of prices which allows it, *the prices used by each couple of operators being specific to them.* The distribution of the realized surplus between operators is determined by the system of prices used in the exchanges between them.

14. *Conditions of Equilibrium and Maximum Efficiency*

14.1 *Situations of stable economic equilibrium.* – *In essence all economic operations of whatever type may be considered as reducing to the search for, the achievement of, and the distribution of surpluses.* Thus stable general economic equilibrium exists if, and only if, in the situation under consideration, there is no realizable surplus, which means

$$\delta\sigma_u \leq 0 \qquad (1.12)$$

for all feasible modifications of the economy (Allais, 1943, p. 606-612).

In such a situation the distributable surplus is zero or negative for all possible modifications of the economy compatible with its structural relations, and it is impossible to find any set of prices that would permit effective bilateral or multilateral exchanges (accompanied by the implied production operations) which are advantageous to all the operators concerned.

14.2 *Situations of maximum efficiency.* – A situation of maximum efficiency can be defined as a situation in which it is impossible to improve the situation of some people without undermining that of others, i.e. to increase certain preference indices without decreasing others.

In the hyperspace of preference indices, a field corresponding to a set of feasible situations and a field corresponding to unrealizable situations can both be defined. *The set of states of maximum efficiency represents the boundary between the possible and the impossible* (Fig. 1).

SITUATIONS OF MAXIMUM EFFICIENCY
IN THE SPACE OF PREFERENCE INDICES
ILLUSTRATIVE DIAGRAM

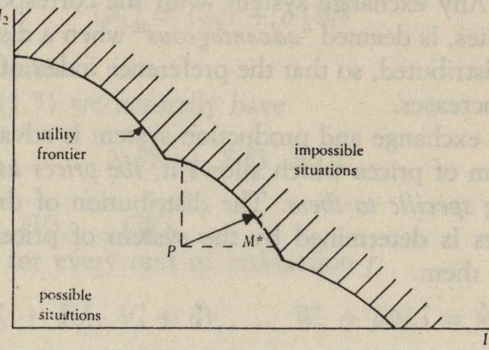


FIGURE 1

14.3 *Equivalence Theorems* (*I*) *and* (*II*). – From the definitions of the situations of maximum efficiency and stable general economic equilibrium, it follows, *with the greatest generality and without any restrictive hypothesis of continuity, differentiability or convexity*, except naturally the continuity hypothesis for the common good (U), that:

– Any state of stable general economic equilibrium is one of maximum efficiency (*First theorem of equivalence*)

– Any state of maximum efficiency is one of stable general economic equilibrium (*Second theorem of equivalence*).

Since *there can be no stable general economic equilibrium if there is any distributable surplus, every state of stable general economic equilibrium is a state of maximum efficiency.*

Conversely, if there is maximum efficiency, there is no realizable surplus which could be used to increase at least one preference index without decreasing the others, and consequently, *every state of maximum efficiency is a state of stable general economic equilibrium.*

Because of the theorems of equivalence, the terms "*conditions of stable general economic equilibrium*" and "*conditions of maximum efficiency*" are used interchangeably below.

15. *The Dynamic Process of the Economy: Decentralized Search for Surpluses*

15.1 *The decentralized search for surpluses and the model of an economy of markets. – In their essence all economic operations, whatever they may be, can be thought of as boiling down to the pursuit, realization and allotment of distributable surpluses.* The corresponding model is *the 1967 Allais model of the economy of markets,* defined by the fundamental rule that every operator tries to find one or several other operators ready to accept at specific prices a bilateral or multilateral exchange (accompanied by corresponding production decisions) which will release a positive surplus *that can be shared out, and which is realized and distributed once discovered.* Thus the evolution of the markets economy is characterized by the condition

$$\delta I_i \geq 0 \tag{1.13}$$

for every consumption unit.

15.2 *Theorem of convergence (III).* – Since in the evolution of an economy of markets, surpluses are constantly being realized and allotted, the preference indices of the consumption units are never decreasing, at the same time as some are increasing. This means *that for a given structure, that is to say, for given psychology, resources, and technical know-how,* the working of an economy of markets tends to bring it nearer and nearer to a state of stable general economic equilibrium, and this state is a state of maximum efficiency.

Naturally such evolution takes place only if a sufficient information exists about the actual possibilities of realizing surpluses.

To any given initial situation whatsoever, assumed not to be a situation

PROCESS OF DYNAMIC EVOLUTION
ILLUSTRATIVE DIAGRAM

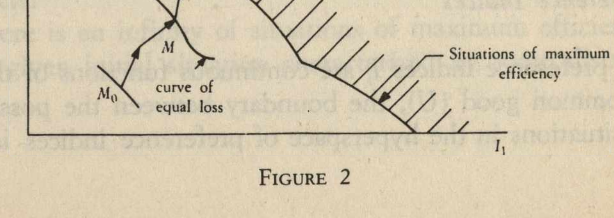


FIGURE 2

of equilibrium and represented by a point such as M_o, there corresponds an infinite number of feasible paths M_o M^* which may lead to a situation of equilibrium M^*, and along each of which no preference index is decreasing (Fig. 2).

15.3 *Economic loss.* – The loss σ_u^* which is associated with a given situation is defined *as the greatest quantity of the good* (*U*) *which can be released in a transformation of the economy for which all the preference indices remain unchanged* (Fig. 2) (Allais, 1943, p. 638-649).

It is a well determined function

$$\sigma_u^* = F[I_1, I_2, ..., I_n, U_o, V_o, ..., W_o] \qquad (1.14)$$

of the preference indices I_i *and of the resources* V_o *which characterize this situation.* The loss σ_u^* is an indicator of inefficiency, and $-\sigma_u^*$ an indicator of the efficiency *of the economy as a whole.*

The loss is minimum and nil in every state of maximum efficiency, and positive in every feasible situation which is not a state of maximum efficiency. It decreases in any modification of the economy, whereby some preference indices increase, others remaining unchanged, or whereby some surpluses are released with no decline in some preference indices.

15.4 *The changing structure of the economy.* – As psychological patterns vary, as techniques are improved, or as new resources are discovered (or existing resources depleted), the set of situations of maximum efficiency relative to the indices of preference constantly undergoes change over time.

Consequently, situations of equilibrium and maximum efficiency are never reached, and what is really important is to determine the rules of the game which must be applied to come constantly closer to them as rapidly as possible.

At a given time *t*, *if* information is sufficient and *if* the adjustments are sufficiently rapid, the point representing the economy will never be very far from the maximum efficiency surface of that time *t*.

16. *Representation of the States of Maximum Efficiency in the Hyperspace of the Preference Indices*

As the preference indices I_i are continuous functions of the quantities U_i of the common good (*U*), the boundary between the possible and the impossible situations in the hyperspace of preference indices is constituted by a continuous surface. On this surface the loss σ_u^* is nil. This representation permits to demonstrate immediately, by simple topological considerations, propositions the demonstration of which could be otherwise very difficult. (The paternity of this representation has been unduly attributed to P. Samuelson, 1950, whereas it was published for the first time in Allais, 1943, and systematically used by Allais the following years; see Allais, 1971, n. 11, p. 385; and 1974 A, n. 18, p. 176-177).

In the general case, the surface of maximum psychological possibilities has a complex form which cannot be specified *a priori*. To illustrate *Figure 1* represents in the case of a *particular model* and of two preference indices, the maximum efficiency curve, i.e. the boundary between the possible and the impossible (see Allais 1974 B, p. 630).

For every feasible situation which is not a state of maximum efficiency, represented by a point such as P, there are an infinity of realizable displacements PM enabling a situation of maximum efficiency M* to be approached, such that all the preference indices have greater values than in the initial situation P* (Fig. 1).

Figure 2 presents an illustration of the process of dynamic evolution *by releasing and sharing out of surpluses* during which the loss σ_u^* is constantly decreasing.

17. *General Comment*

A *markets economy* can be defined as one in which the operators – consumption, production, and arbitrage units – coexist and are free to undertake any exchange transaction or production operation *which can result in rendering some distributable surplus available*. The principle of the markets economy is that any surplus realized is shared among the operators involved.

How the surpluses achieved are shared out depends on the system of prices used in the exchanges between the operators concerned. The prices used are always *specific* to the exchange and production operations considered and *there is never a unique system of prices* used by all operators.

The diagrammatic representation on Figures 1 and 2 reveals clearly three basic facts:

1) There is an infinity of situations of maximum efficiency corresponding to a given initial situation characterized by some distribution of property.

2) To each situation of maximum efficiency there corresponds a final distribution of property.

3) This final distribution depends on the initial situation and the distribution of surpluses in the course of the transition.

Thus there is *a very strong interdependence between the point of view of efficiency corresponding to the discovery and realization of surpluses and the ethical point of view corresponding to their sharing.*

In any event, since only what is produced can be shared, the incentive stemming from the partial or total appropriation of the surpluses by operators appears as a fundamental factor for the functioning of the economy of markets.

18. *External Effects*

The displacements ($\delta\mathcal{E}$) of the economy considered in § 13 naturally integrate the external effects which appear both in the preference functions of the consumption units and in the production functions. These can have a major effect on the preference functions corresponding to the state, the provinces and towns.

II. – THEORETICAL FOUNDATIONS OF THE EQUIMARGINAL PRINCIPLE WHEN CONTINUITY AND DIFFERENTIABILITY CAN BE ASSUMED

21. *The Hypotheses of Continuity and Differentiability*

The preceding definitions and theorems are very general and do not make any hypothesis of continuity, derivability or convexity, except the hypothesis of continuity for the common good (U).

We now assume in addition only that all the quantities and functions considered are continuous and that all functions have first and second order derivatives, the following developments being *totally independent of any hypothesis of general convexity.*

21.1 *Derivatives.* – From the conventions of signs adopted earlier (§ 12), it follows that for any i, j and V

$$f'_{iv} = \partial f_i / \partial V_i \geq 0 \qquad f'_{jv} = \partial f_j / \partial V_j \geq 0 \tag{2.1}$$

The second partial derivatives are marked

$$f''_{ivw} = \partial^2 f_i / \partial V_i \partial W_i \qquad f''_{jvw} = \partial^2 f_j / \partial V_j \partial W_j \tag{2.1*}$$

In the following, the symbol $\overline{d^2 g}$ represents the second differential

$$\overline{d^2 g} = \sum_u^w g''_{v^2}\, dV^2 + 2 \sum_{u,\, v} g''_{vw}\, dV\, dW$$

of a function $g\,(U,\ V,\ \ldots,\ W)$ when all parameters in that function are considered as independent, while the symbol $\overline{d^2 g_u}$ represents what this second differential becomes after dU has been replaced by its expression derived from

$$dg = \sum_u^w g'_v\, dV = 0$$

(Allais, 1968 A, Vol. II, p. 77-78; 1973 B, p. 151-155; 1981, p. 688-689).

21.2 *Convexity and concavity*. – The local properties of diminishing or increasing marginal returns are related to the local conditions of convexity or concavity. Convexity is defined as follows:

Ordinal Fields of Preference: A field of choice is said to be convex in the whole space (*postulate of general convexity*) if, at all points of the field, the condition

$$I\,(M_0) \leq I\,(M_1)$$

entails

$$I\,(M_0) \leq I\,(M)$$

with

$$M = \lambda M_0 + (1 - \lambda)\, M_1 \qquad 0 < \lambda < 1$$

There is *local convexity* at M_0 if this condition is only satisfied for

$$|\overrightarrow{M_0 M_1}\,| < \varepsilon$$

where ε is a given positive number.

When differentiability is assumed local convexity implies

$$\overline{d^2 f_{iu}} \leq 0 \qquad \text{for} \qquad df_i = 0 \tag{2.2}$$

Fields of production: A field of production is said to be convex over the whole space (*postulate of general convexity*) if, for any two possible points M_0 and M_1, the centre of gravity defined by the relation

$$\overrightarrow{M} = \lambda \overrightarrow{M_0} + (1 - \lambda) \overrightarrow{M_1}$$

is likewise a possible point for

$$0 < \lambda < 1$$

Local convexity obtains at M_0 if the preceding condition is only satisfied for

$$|\overline{M_0 M_1}| < \varepsilon$$

where ε is a given positive number.

When differentiability is assumed, local convexity implies

$$\overline{d^2 f_{ju}} \leq 0 \qquad \text{for} \qquad df_j = 0 \tag{2.2*}$$

21.3 *General structure of the fields of choice and production. – In fact there is no production operation that does not begin by providing increasing marginal returns, and it is only beyond a certain threshold that diminishing marginal returns are observed. That is a general physical law of nature.* (Allais, 1943, p. 193-195; 1968 A, II, p. 68-96; 1971, p. 362-364; 1974 A, p. 153-157).

Similarly it can be considered as an introspective datum that psychological returns begin by increasing but in the end always decrease beyond certain threshold values. That is a *general psychological law.* (Allais, 1968 A, II, p. 109-138; 1971, p. 360-362; 1974 A, p. 153-155).

These are two fundamental properties of fields of choice and production. They rule out the postulate of general convexity which is generally accepted in the contemporary literature.

22. *Generation of Distributable Surplus*

22.1 *General formulation.* – Consider any economic state ($\mathcal{E}$) and a realizable modification ($\delta\mathcal{E}$) such that all the preference indices I_i remain constant (*isohedonous modification*).

Let the conditions of constancy of these indices and the conditions corresponding to the production functions be written in the same general form

$$g_k (U_k, V_k, ..., W_k) = 0 \tag{2.3}$$

where U_k, V_k, ..., W_k represent the consumption of both consumption and production units. *By convention,* any quantity V_k, if positive, represents a consumption, either by a consumption or a production unit. For any production or consumption unit, any parameter V_k, if negative, represents production of a good or a service.

Let dU_k, dV_k, ..., dW_k, be the first order differentials of the variations δU_k, δV_k, ..., δW_k of consumptions U_k, V_k, ..., W_k in the displacement $(\delta\mathcal{E})$. From (2.3), we have

$$g'_{ku} dU_k + g'_{kv} dV_k + ... + g'_{kw} dW_k = 0 \tag{2.4}$$

Let δV_{kl} be the quantity of (V) received by the consumption or the production unit k from the consumption or production unit l. By definition, we have

$$\delta V_k = \sum_{k \neq l} \delta V_{kl} \tag{2.5}$$

$$\delta V_{lk} = - \delta V_{kl} \tag{2.6}$$

Assuming that the displacement $(\delta\mathcal{E})$ is such that

$$\sum_k \delta V_k = 0, ..., \sum_k \delta W_k = 0 \tag{2.7}$$

Let

$$\varepsilon^k_{v,u} = g'_{kv}/g'_{ku} \tag{2.8}$$

The ratio $\varepsilon^k_{v,u}$ is the *coefficient of marginal equivalence (or marginal rate of substitution)* of goods (V) and (U) for operator k (Allais, 1943, p. 609-610, and 617-621).

From (2.4) and (2.8), we have the relation

$$dU_k = - [\varepsilon^k_{vu} \, dV_k + ... + \varepsilon^k_{wu} \, dW_k] \tag{2.9}$$

between the first order differential dU_k, dV_k, ..., dW_k.

If dU_k is positive, operator k receives a quantity dU_k to within the second order. If dU_k is negative, operator k supplies a quantity $- dU_k$ to within the second order.

From the condition (2.7), *it results that the displacement considered releases a global distributable surplus*

$$\delta\sigma_u = - \sum_k \delta U_k \tag{2.10}$$

representing the excess of the quantities *supplied* over the quantities *received* of good (U) whose first order differential is

$$d\sigma_u = -\sum_k dU_k \tag{2.11}$$

From (2.5) and (2.9)

$$dU_k = -\sum_v^w \left[\varepsilon^k_{vu} \sum_{\substack{k,l \\ k<l}} dV_{kl}\right] \tag{2.12}$$

and from (2.6), we have (Allais, 1952 C, p. 31; 1968 A, II, p. 174; 1981, p. 88)

$$d\sigma_u = \sum_v^w \sum_{\substack{k,l \\ k<l}} (\varepsilon^k_{vu} - \varepsilon^l_{vu})\, dV_{kl} \tag{2.13}$$

According to definition (1.11) $d\sigma_u$ is *the first differential of the global distributable surplus* $\delta\sigma_u$ *released in the displacement considered.*

For all economic agents the unit of value is defined by condition $u_k = u = 1$. The marginal values v_k ... w_k of goods (V) ... (W) for unit k are defined with respect to the u_k by the relations

$$\frac{g'_{ku}}{u_k} = \frac{g'_{kv}}{v_k} = \ldots = \frac{g'_{kw}}{w_k} \tag{2.14}$$

$$u_k = u = 1 \tag{2.14*}$$

Under the adopted convention for signs, all the v_k are positive. We have from (2.8) and (2.14*)

$$\varepsilon^k_{vu} = v_k \tag{2.15}$$

and relation (2.13) is written

$$d\sigma_u = \sum_v^w \sum_{\substack{k,l \\ k<l}} (v_k - v_l)\, dV_{kl} \tag{2.16}$$

where v_k and v_l are the marginal values of good (V) for units k and l. This summation covers *all* operators, consumption and production units. *It can thus be seen that all the differences between the marginal values in the situation* $(\mathcal{E})$ *can give rise to the release of potential surpluses which can be released and distributed.*

The meaning of relation (2.16) is immediate. Thus if $v_k > v_l$ the relative value of good (V) is higher for operator k than for operator l. The transfer of a positive quantity dV_{kl} of good (V) from operator l to operator k therefore creates an additional *positive* value

$$d\sigma_{ukl} = (v_k - v_l)\, dV_{kl} \tag{2.16*}$$

If in the "*isohedone*" transformation considered surpluses are released, all positive, they can be distributed in such a way as to increase all preference indices. In such a modification of the economy, the maximum distributable surplus diminishes, and *the point representing the economic situation considered moves closer to the surface of maximum efficiency in the hyperspace of preference indices. Naturally, for this condition to obtain, the corresponding exchanges and the changes of the consumptions and productions they imply in the production system, must effectively occur.* We then have (Allais, 1981, p. 53)

$$d\sigma_u = \sum_i dI_i/I'_{iu} \tag{2.17}$$

where the dI_i are the first differentials of the preference indices I_i with, for any i (condition 1.13 above)

$$dI_i > 0 \tag{2.18}$$

When all the marginal values, i.e., all the coefficients of equivalence are equal, the first differential of distributable surplus is nil, and the first order conditions of a situation of equilibrium and maximum efficiency are met. Figure 2 above provides a geometrical representation of this process in a two-dimensional space.

22.2 *Psychological values and marginal psychological values.* – Naturally, the v_k are only marginal values for the operators. The psychological values v_i^* of the consumption V_i of a subject i is defined by the relation

$$f_i\,(U_i + v_i^* V_i,\ 0,\ \ldots,\ W_i) = f(U_i,\ V_i,\ \ldots,\ W_i) \tag{2.19}$$

where $v_i^* V_i$ is the sum he would have to be paid to offset the drop in his consumption V_i to zero. The unit value v_i^* is generally much higher than the marginal value v_i corresponding to relations (2.14) and (2.15).

In any event, a consumption is only advantageous when its psychological values is higher than its marginal value, because, if this were not so, it would be in the subject's interest to reduce his consumption V_i.

22.3 *Theory of surpluses and marginal analysis.* – A single relation, the relation (2.16) (or the equivalent relation 2.13) *condenses the whole margi-*

nal approach as it has developed for over a century. Subject only to the hypotheses of continuity and derivability implied by any marginal theory, it applies in all cases, and its simplicity is really striking.

This relation also shows that equilibrium and maximum efficiency can only obtain when all marginal values are equal.

23. *Conditions of Stable General Economic Equilibrium and Maximum Efficiency of the Economy*

From the foregoing (§ 14) it follows that *the necessary and sufficient condition for a situation* ($\mathcal{E}$) *to be of stable equilibrium and maximum efficiency is that the distributable surplus* $\delta\sigma_u$ *defined by* (1.10) *and* (1.11) *be negative or zero i.e.*

$$\delta\sigma_u \leq 0 \tag{2.20}$$

for every feasible modification ($\delta\mathcal{E}$), *i.e., every modification that is compatible with the constraint conditions, i.e., the structural relations of the economy.*

Condition (2.20) implies the two conditions (Allais, 1943, p. 612)

$$d\sigma_u = 0 \tag{2.21}$$

$$d^2\sigma_u \leq 0 \tag{2.21*}$$

for any realisable and reversible modification ($\delta\mathcal{E}$) in which the expressions of $d\sigma_u$ and $d^2\sigma_u$ represent the first and second differential of $\delta\sigma_u$.

Actually, *the first order condition* $d\sigma_u = 0$ implies that when the quantities V_k are not nil, all the marginal values v_k are equal to a same value v and a same system of prices *u, v, ..., w then exists for all the operators concerned, such that*

$$\frac{g'_{ku}}{u} = \frac{g'_{kv}}{v} = \ldots = \frac{g'_{kw}}{w} \tag{2.22}$$

These equalities condense the general equimarginal principle into a single formulation. They express the fact that in a situation of equilibrium and maximum efficiency, the psychological (or objective) value v_k of the last dollar is the same, for any operator (consumption or production unit), whatever use it is put to.

For the quantities V_k which are nil (terminal equilibria), we necessarily have

$$v_k \leq v \tag{2.23}$$

since, if this were not true, the operator's interest would be to increase V_k from the value $V_k = 0$; he could indeed do this because of the existence of other operators who are in a situation of tangential equilibrium for good (V).

The *second order condition* (2.21*) is written using the notation of § 21

$$d^2\sigma_u = \sum_i \frac{\overline{d^2 f_{iu}}}{f'_{iu}} + \sum_j \frac{\overline{d^2 f_{ju}}}{f'_{ju}} \leq 0 \qquad \text{for} \qquad d\sigma_u = 0 \tag{2.24}$$

Expression (2.24) holds whether or not the df_i are equal to zero. It is only subject to the constraint (2.21). If we consider only the modifications of the economy involving units k and l, condition (2.24) is written

$$d^2\sigma_u = \frac{\overline{d^2 f_{ku}}}{f'_{ku}} + \frac{\overline{d^2 f_{lu}}}{f'_{lu}} \leq 0 \qquad \text{for} \qquad d\sigma_{uk} + d\sigma_{ul} = 0 \tag{2.24*}$$

which shows that one unit k or l at most is in a situation of local concavity (Allais, 1968 A, p. 196-199; 1974 A, n. 125, p. 184; 1981, p. 65).

According to (2.2) and (2.2*), condition (2.24) thus shows that *when in a situation of maximum efficiency consumption or production units consume (or produce) the same goods, one unit at most is in a situation of local concavity, i.e., in a situation of marginal increasing returns.*

Consequently, when maximum efficiency obtains, most operators are in a situation of local convexity and marginal decreasing returns. However, this condition cannot be interpreted as meaning that all fields of choice and production are convex everywhere, *this hypothesis being totally contradicted by observed data* (§ 21.3).

When local convexity obtains for a consumption unit, its index of preference is effectively at a maximum, subject to the budgetary constraint, equilibrium prices being taken as given. Similarly if local convexity obtains for a production unit, the unit's income is effectively at a maximum, equilibrium prices again being taken as given. However, these two principles, which in any case could be valid only for a situation of maximum efficiency, cannot be considered as corresponding in all cases to optimum behaviour, and they cannot be taken to be of general value. As a matter of fact and for instance, if, in a situation of maximum efficiency, a production unit is in a situation of local concavity, its income is *minimum*, the equilibrium prices being considered as given (see § 41.4 below).

According to (2.17) and (2.21) we have

$$d\sigma_u = \sum_i d\sigma_{ui} = \sum_i dI_i/I'_{iu} = 0 \tag{2.25}$$

in a situation of equilibrium and maximum efficiency.

Conditions (2.22) and (2.24) show *the total symmetry of the implications of the psychological and technical structures of the economy.*

Conditions (2.21 and 2.22) naturally represent only local conditions. Only if condition (2.20) holds can it be taken that the situation is one of maximum efficiency in general, and not merely a purely local state of maximum efficiency.

24. *Approximate Value of the Economic Loss Corresponding to the Non Equality of Marginal Values in the Neighbourhood of a Situation of Maximum Efficiency*

The integration of equation (2.16) along a path leading to a state of maximum efficiency leads to the following approximate estimate to within third order accuracy of the global loss involved in the initial situation

$$\sigma_u^* \sim \frac{1}{2u} \sum_v \sum_{\substack{k,l \\ k<l}} (v_k - v_l)\, \delta V_{kl}^* \tag{2.26}$$

In this relation the *quantities $v_k - v_l$ represent the differences of marginal values in the initial state considered, and the δV_{kl}^* are the quantities of the good* (*V*) *received by operator k from operator l in the transition from the initial to the final state. Relation* (2.26) *is of the broadest generality, and holds whatever the initial state* (Allais, 1952, p. 31-32, n. 8; 1968 A, II, p. 207; 1981, p. 110).

Its simplicity is really striking in view of the complexity of the concept it represents, namely the maximum of the distributable surplus for all the modifications which the economy can undergo while leaving the preference indices unchanged.

In the neighbourhood of a situation of maximum efficiency, the $(v_k - v_l)$ *and δV_{kl}^* are first order quantities, whereas the loss σ_u^* is only of the second order.* However, since the δV_{kl}^* are of the first order, the variations δI_i of the preference indices are also of the first order. As a result, in the neighbourhood of a situation of maximum efficiency, taxes have major first order effects on the distribution of income but only second order effects on the efficiency of the economy. This also explains how a country, by implementing an appropriate tax system on imports or exports, can increase its standard of living relatively to the rest of the world. International efficiency is reduced, but the standard of living of that country is increased.

Consequently also, if there are relatively small differences between marginal values, the gains originating from exchanges are much lower than those corresponding to an improvement of available techniques by the production units. *This is a basic finding with many applications.* For instance, it can be deduced that if there are no great differences in relative prices, the gains expected from the liberalization of international trade are only of the second order, and in any event, quite low by comparison with those stemming from the growing incentive to use more effective technologies (i.e., the improvement of the techniques represented by the production functions) originating from the pressure of competition caused by the liberalization of exchanges.

At all events, relation (2.26) *shows that any difference between marginal values corresponds to a loss of efficiency.* This is a general principle of which the well-known saying: "everything sold below cost is wasted", is only a special illustration.

Relation (2.26) also shows that a uniform tax system on consumption goods does not jeopardize efficiency and only entails a transfer of income to the State.

25. *Imputation*

If inputs $X, Y, ..., Z$ and outputs $A, B, ..., C$ are given in explicit form in a production function

$$F(A, B, ..., C, X, Y, ..., Z) = 0 \qquad (2.27)$$

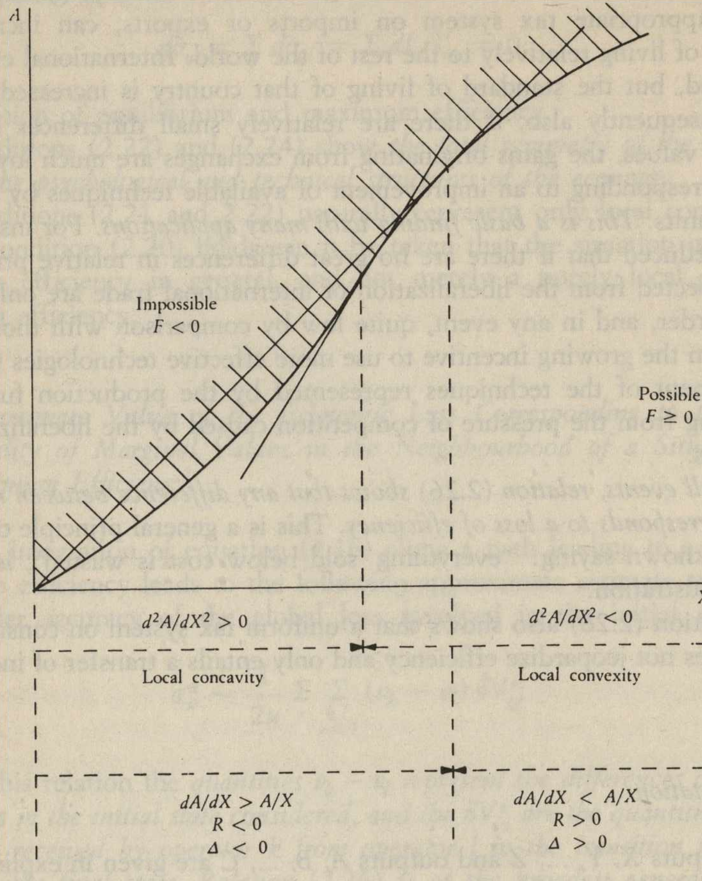
$$B > 0; \; Y > 0; \; F'_B < 0; \; F'_Y > 0$$

then, given the properties of the fractions, the equations (2.22) corresponding to conditions of equilibrium and maximum efficiency are written

$$\frac{-F'_A}{a} = ... = \frac{-F'_C}{c} = \frac{F'_X}{x} = ... = \frac{F'_Z}{z} = \frac{-\Sigma AF'_A - \Sigma XF'_X}{\Sigma aA - \Sigma xX} \qquad (2.28)$$

AVERAGE YIELD, MARGINAL YIELD, CONVEXITY AND INCOME

ILLUSTRATION



Legend

$$F(\mathrm{A},\ \mathrm{X}) \geq 0 \qquad A > 0 \qquad X > 0$$

$$-F'_A > 0 \qquad F'_X > 0 \qquad dA/dX = -F'_X/F'_A$$

$$\Delta = -AF'_A - XF'_X \qquad R = aA - xX$$

$$-\frac{F'_A}{a} = \frac{F'_X}{x} = \frac{-AF'_A - XF'_X}{aA - xR} = -XF'_A\frac{\dfrac{A}{X} - \dfrac{dA}{dX}}{R}$$

FIGURE 3

Let

$$\Delta = - \Sigma A F'_A - \Sigma X F'_X \tag{2.29}$$

$$R = \Sigma a A - \Sigma x X \tag{2.30}$$

in which Δ is a local structural indicator and R the income of the production unit *calculated at equilibrium prices.* It can be seen that R is positive, nil or negative as Δ is positive, nil or negative (Allais, 1943, p. 574-577; 1968 A, p. 84-87).

To simplify, consider the case in which a single output A is produced from a single input X (Figure 3). The indicator Δ, and therefore R, is positive, nil or negative according to whether the marginal yield $dA/dX = - F'_X/F'_A$ is inferior or superior to the average yield A/X. The first case corresponds to a situation of decreasing *marginal* return, the second to one of increasing *average* return. In the first case, there is local convexity (§ 21.2). In the second, there may be local convexity or concavity (Allais, 1981, p. 350).

These properties are easily extended when the production function is represented by the function (2.27). In the first case (diminishing marginal returns) the global expenditure corresponding to the reward accruing to each factor of production according to its marginal productivity is inferior to the global value of the joint products calculated at their marginal values, and the income R is positive. In the second case (increasing average returns), there is a deficit; but the deficit exists only insofar as it is postulated that every unit produced is sold at the same price (§ 41 below).

26. *The General Theory of Surpluses as a Generalization of the Theory of Marginal Equivalences*

In the light of the above, classical marginal theory is clearly only a special case of a much more general theory: the general theory of surpluses (Part I). Whereas the classical marginal theory assumes continuity and differentiability, the only hypothesis underlying the general theory of surpluses is the hypothesis of continuity of the quantities of good (U) (§ 11). *In the final analysis, the general theory of surpluses extends the theory of marginal equivalences to the most general case of finite variations.*

III. – MARGINAL EQUIVALENCES AND THE APPROACH FOR CONDITIONS OF EQUILIBRIUM AND MAXIMUM EFFICIENCY

31. *The Equimarginal Principle*

The *equimarginal principle* is the proportionality of marginal desirabilities f'_{iv} and marginal productivities f'_{jv} (relation 2.22) under the same system of prices in situations of general economic equilibrium and maximum efficiency. This means that in such a situation, the subjective and objective rates of substitution between any two goods are equal for all consumption and production units and these subjective and objective ratios are equal to each other.

This is effectively true when continuity and differentiability hold, but it cannot be deduced, as is too frequently done, that it is an operational rule whose systematic application would allow a situation of maximum efficiency to be reached. Such a situation never obtains, and this property is proven only for situations of maximum efficiency. *The equimarginal principle is therefore not a usable operational rule. It informs us about the conditions obtaining in situations of maximum efficiency. It does not show how to reach them.*

The only relation that can be used is relation (2.13) or the equivalent relation (2.16), which shows that a surplus can be achieved wherever differences in marginal equivalences exist.

Furthermore, there is no unique situation of maximum efficiency that can be reached from a given initial state. There is an infinite number of such situations (Figures 1 and 2), and even if it were possible to specify one, the corresponding system of prices would remain unknown and what would be at issue would be to determine it, or at least to come as close as possible to it.

Thus, knowledge only of the general characteristics of a situation of maximum efficiency could not inform us as to the path to be followed to reach it.

32. *Approach of a Situation of Maximum Efficiency from the Consideration of Marginal Equivalences*

The analysis presented above (§ 22) allows to specify the process to be implemented to come closer to situations of maximum efficiency.

32.1 *The search for potentially available surpluses.* – Surpluses can be released for three reasons: – Some available resources are left unused; – The techniques used in the production sector may not be the most efficient possible; – Advantageous exchange transactions and the productions they imply may fail to occur.

In the third case in which the production techniques are assumed to be the most efficient, the differences in marginal values can serve as a guide in seeking for realizable surpluses (relation 2.16), but, naturally and in particular, only if calculations of marginal productivities f'_{jv} have meaning, which implies that the production functions are indeed the most efficient in the sense of § 12.2.

32.2 *Realization of potentially available surpluses.* – In the case of the differential approximation, it can be shown that any exchange and production system, such that for every consumption unit i and production unit j involved we have

$$\sum_V v_i dV_i > 0 \qquad (3.1)$$

$$\sum_V v_j dV_j > 0 \qquad (3.2)$$

releases a surplus and brings the economy closer to the maximum efficiency surface. Therefore, whatever the specific systems of prices used in a system of exchanges, if, *for each operator*, the global value of what is received in the exchange calculated with this operator's *system of marginal values* is positive, the corresponding system of exchanges (with the production operations it implies) appears to be advantageous from the point of view of efficiency of the economy as a whole.

Consequently, as regards marginal modifications, economic calculation by operators is possible even when there is no common system of prices for all operators. An operation can increase the efficiency of the economy if it generates a value for each operator involved *and is effectively realized.*

When the economy as a whole is not in a situation of maximum efficiency, but when there is a system of marginal values common to some consumption and production units, any decision relating to exchange or production which can create value, at prices which are marginal values common to the units considered, *brings the economy closer to the situations of maximum efficiency providing the decision is effectively implemented.*

32.3 *Sharing of surpluses.* – In sum, the effect of any exchange operation, with the corresponding production operations it implies, which allows a surplus to be distributed *so as to improve every participant's situation*, is to reduce the maximum distributable surplus, and therefore, to increase the efficiency of the economy (Figure 2).

The final situation of maximum efficiency to which the economy will finally be brought closer, will basically depend on the distribution of surpluses, which will be obtained by all those who find and release them. It is mainly this sharing of surpluses in the course of an operation which determines, more often than the initial state, the final situation which the economy will approach (Figure 2).

The appropriation of the surpluses realized by the operators concerned is a *major stimulus* for them to seek these surpluses and act to ensure that they are achieved.

33. *Limits of Application of the Equimarginal Principle*

In any event, the *equimarginal principle* can only apply under very restrictive conditions.

33.1 *Terminal equilibria.* – The equimarginal principle is naturally no longer valid for situations in which there is no tangential equilibrium. In this case, the principle of equality of marginal values must be replaced by the principle that the marginal value is lower than or equal to the equilibrium price (condition 2.23).

33.2 *Lack of knowledge of preference and production functions.* – In general, the preference and production functions f_i and f_j are unknown. Knowing them over their whole field of definition is unnecessary, because in all cases the approach of the economic agents is always to select amongst a limited number of choices. Conditions (2.22) provide an accurate representation of the situation towards which the economy is tending, but the actual process which can fulfill them is very different. The consideration of these conditions is therefore very useful for economic analysis, but generally remain non-operational. *The only process to be considered corresponds to the release of potentially available surpluses.*

33.3 *Joint products.* – In the case of joint products, it is very difficult, if not impossible, to calculate marginal costs. For instance, this is generally true of overall costs or of the user value of a durable good over time.

33.4 *Indivisibilities.* – The equimarginal principle is only valid if the quantities and functions are continuous. But this is almost never true. For instance if I buy a television set, the quantity is *zero or one*, and nothing else; the same remark also applies to the purchase of a house.

The hypothesis of continuity currently accepted in the literature is only a hypothesis. In many cases it can be viewed as valid, at least as a first approximation, and in conjunction with the assumption of differentiability of functions with which it can be associated in practice, it provides suggestive analyses. However, in general, it cannot be taken as a rigorous hypothesis. The hypothesis of continuity is even totally unacceptable in some major cases (buildings, dams, motorways, etc...).

33.5 *The general criterion of distributable surplus.* – When the differential calculus no longer applies, even as a first approximation as in particular in the case of marked indivisibilities, the equimarginal principle no longer holds, and another much more general principle whose application is more complex must be used to replace it, i.e., the principle of absence of any positive realizable surplus in a situation of maximum efficiency (§ 14).

34. *Durable Goods, the Unpredictability of the Future, and Marginal Equivalences over Time*

The theory above only applies *to constant psychological and technological structures, considered within a given time period, and it assumes that there are no durable goods.*

The consideration of durable goods introduces *two basic difficulties.* The first is that durable goods providing services over time are *indivisible* in most cases. The second *major difficulty* is that *the future is unpredictable,* and that forecasts are inevitably affected by some uncertainty.

The marginal analysis can only be extended under certain conditions. The simplest model to be considered, at least as a first approximation, is that involving *perfect foresight.*

34.1 *Case of general equilibrium over time under the assumption of perfect foresight. – If there is a perfect foresight,* stable general equilibrium over time implies similar conditions to conditions (1.5), (1.12), (2.22) and (2.24) with additional conditions.

The first of these are structural conditions implying that for each durable good (V) used from period T_{q+1}, to period T_{q+s}, we have

$$V_{q+1} = \dots = V_{q+r} = \dots = V_{q+s} = \bar{V}_q \tag{3.3}$$

where the $\bar{V}_q$ and $V_{q+1} \dots V_{q+s}$, using suitable units, represent respectively the quantity of this good produced over period T_q and the services rendered by its use over periods $T_{q+1} \dots T_{q+s}$.

The second conditions are written

$$\bar{v}_q = v_q + \dots + v_{q+r} + \dots + v_{q+s} \tag{3.4}$$

where $\bar{v}_q$ is the discounted marginal value of the durable good during its production period and the v_{q+r} are the discounted marginal values of the services V_{q+r}. For each durable good, they express *the equality of the marginal production cost of this good and the sum of the discounted marginal values of its future services* (Allais, 1943, p. 344-346, and 619-620; 1968 A, p. 198-199; 1981, p. 156).

The equimarginal principle is reflected here by relation (3.4) the form of whose expression is very different from that of relations (2.22), because the structural conditions (1.5) are replaced here by the structural conditions (3.3).

If the continuous interest rate $i(t)$ is used, the discounting principle can be written in continuous notation

$$\bar{v}(t) = \int_t^{\infty} v(\tau)\, e^{-\int_t^{\tau} i(u)\,du}\, d\tau \tag{3.5}$$

whose differential expression is written

$$-\,d\bar{v}(t)/dt + i(t)\,\bar{v}(t) = v(t) \tag{3.6}$$

where $\bar{v}(t)$ and $v(t)$ represent respectively the value of the durable good considered and its user value (Allais, 1943, p. 361-362).

The user value v corresponds to the conditions of maximum efficiency at time t. In the case of a perfect foresight, according to (3.6), it is equal to the sum of the interest on capital at that time and amortization. *User value can thus be interpreted in terms of cost, but this interpretation is only possible in the case of perfect foresight and maximum efficiency over time* (Allais, 1964, § 20, p. 4-5).

In a world of perfect foresight but not in equilibrium, output of a durable good can release a surplus if its global cost is lower than the sum of the discounted global psychological values of its future services (§ 22.1). There can only be equilibrium, and therefore maximum efficiency, when output has grown so far that the corresponding marginal values are equalized (Condition 3.4). There is therefore a double "rentability": a global and a marginal "rentability" (Allais, 1964, § 13-19).

In any event, if perfect foresight is assumed, the analysis of the process of equilibrium over time brings into light circumstances that are altogether new, allowing on the one hand for the dependence of present preference functions in relation to future preference functions, and on the other, allowing not only for the future as well as the present generations. Here the marginal principle of unicity of pure interest rates is no longer a necessary condition for maximum efficiency (Allais, 1947A, p. 162-167, 702-706, 710-718 and 757-759).

Naturally a stationary process is the simplest case of equilibrium over time under perfect foresight. In this case, each period repeats its predecessor, and all forecasts are verified. Relations (3.3) and (3.4) are fully valid here.

The *hypothesis of perfect foresight* has been considered *explicitly* for the first time in the literature by Allais (1943, p. 34-35; and p. 56). Its interest is at the same time to give a *theoretical support* to the formulation of the discounted values (relations 3.5-3.6) and to show clearly the *validity conditions* of this formulation.

If foresight is imperfect, relations (3.4) and (3.6) no longer hold and the user values of a durable good over time are *completely dissociated from its cost of production*.

34.2 *Intertemporal dynamics and imperfect foresight.* – The analysis of an imperfect foresight raises *major difficulties*. The preference and production functions here change over time more or less unpredictably.

The previous model (Parts I and II) can consequently only be used for a period T which is short enough for the imperfection of forecasting to entail only minor effects. The model shows what would occur for unchanged psychology and technology. It is not valid any longer, at least to a large extent, when psychological and technological structures change.

Admittedly consumption units can try to maximize their preference indices in the light of the instantaneous constraint conditions, but the uncertainty about the unfolding of future events will induce some into error, perhaps ruin them, whereas others' predictions will be borne out, and enrich them. It can be seen that condition (1.13), which only applies to the period $T(t)$ considered, does not necessarily entail an increase in every preference index I_i from this period $T(t)$ to the next $T(t + T)$.

Although, by nature, some physical data, such as the transport structures system, change quite slowly, others change rapidly, for example, the replacement of electric calculators by electronic calculators. Relations (3.4) and (3.6) can remain usable in part for the former, but may become altogether invalid for the latter.

Let $\mathcal{E}(t)$ be the state of the economy at time t, $\mathcal{E}^*(t+\theta, t)$ the state of the economy at time $t+\theta$ expected on average at time t, and $\mathcal{E}(t+\theta)$, the state of the economy which will actually prevail at time $t+\theta$. We thus have a real trajectory $C(t+\theta)$ and a forecast trajectory $C^*(t+\theta, t)$

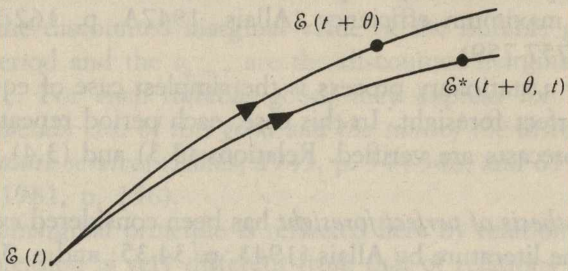


FIGURE 4

At best, both trajectories are tangential at the start and forecasts can be viewed as valid at least for a while. If $\mathcal{E}(t+\theta)$ changes too fast, the two trajectories can move considerably distant from each other.

In the light of such complexity, should we infer that the theory of marginal equivalences is devoid of usefulness? Certainly not. Knowledge of the properties of trajectory $C^*(t+\theta, t)$ envisaged at time t, will make it easier to understand what will occur, and which will only be known ex post, and to act consequently according to whether the psychological and technological structure of the economy will change only a little, or, on the contrary, undergo an upheaval.

In this sense, economic calculation based on marginal equivalences over time remains fundamental, but on its own it is and will always be unable to predict the future. And in some way it is possible to generalize the theory of marginal equivalences to an economy operating under uncertainty, in which the goods are no longer certain goods, but random prospects which are produced and traded following the same principles as those applying to certain goods (Allais, 1952 A).

IV. – ILLUSTRATION OF THE THEORY OF MARGINAL EQUIVALENCES AND ITS GENERALIZATION, THE THEORY OF SURPLUSES: MANAGEMENT OF PRODUCTION UNITS WITH INCREASING RETURNS

Some brief comments on the optimum conditions of management of production units with increasing returns and the transport infrastructure, probably the most difficult problem in the theory of applied economics, *can shed considerable light on the previous analysis.*

When, in a situation of maximum efficiency, increasing returns prevail, global receipts are lower than global expenditure if both are calculated using marginal values, i.e., equilibrium prices (§ 25). This is in particular generally true of railways. Assuming that *all* units are purchased and sold at prices equal to their marginal values, pricing corresponding to this system of prices leads to a deficit. This deficit disappears when pricing is effected at the average costs, but in this case the outcome is a loss (relation 2.26). Hotelling (1938 and 1939) deduces that it is necessary to finance the deficit through subsidies financed out of taxes. *This is the most widely accepted solution in the literature. In reality, the question is much more complicated than it appears at first sight.* (On the criticism of Hotelling's theory, see Allais, 1981, p. 258-294).

41. *A Simple Illustrative Model*

Consideration of a very simple model into which time does not enter can doubtless provide an insight into some basic aspects (Allais, 1968 A, II, p. 248-249).

41.1 *Hypotheses.* – Consider an economy which reduces to a single consumption unit i and a single production unit j. The production function of the production unit is

$$f(A, X) = -A + g(X) = 0 \qquad (4.1)$$

and the preference function of the consumption unit is

$$I = f(A, Y) \qquad (4.2)$$

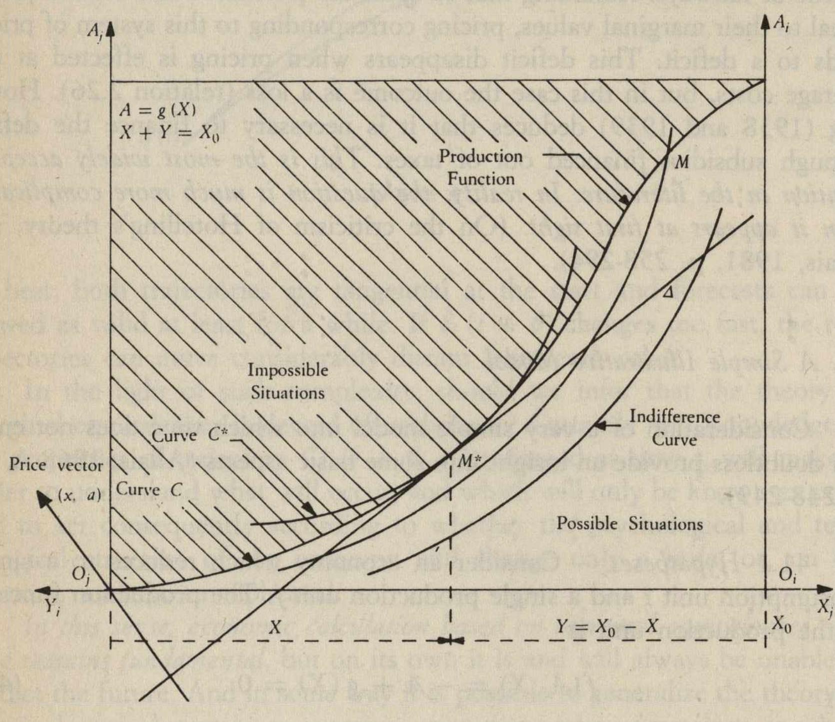
with

$$X + Y = X_0 \qquad (4.3)$$

For example, X can be the amount of work needed to produce A, and Y the leisure left for the consumption unit. X_0 represents the amount of resources available (relation 1.5).

Any possible situation can be represented as in Figure 5. The production function is plotted against the axes (O_jX_j, O_jA_j) and the indifference lines against the axes (O_iY_i, O_iA_i). It is assumed that the relative configuration of the curve C representing the production function and the indiffer-

PRODUCTION UNIT WITH INCREASING RETURN, ASSOCIATED WITH A CONVEX FIELD OF CHOICES
SITUATION OF EQUILIBRIUM AND MAXIMUM EFFICIENCY
ILLUSTRATION



$$\overline{d^2f_{iu}} < 0 \qquad \overline{d^2f_{ju}} > 0$$

$$\frac{\overline{d^2f_{iu}}}{f'_{iu}} + \frac{\overline{d^2f_{ju}}}{f'_{ju}} < 0$$

FIGURE 5

ence lines is as shown in Figure 5. The curve C^* represents the indifference line which is tangential to the curve C. At the point of contact M^*, the index of preference I is at a maximum subject to the constraint condition (4.1).

41.2 *Conditions of equilibrium and maximum efficiency.* – There is *maximum efficiency* at M^* because the index I is maximum under the conditions of constraint (4.1) and (4.3). This maximum implies first and second order conditions corresponding to relations (2.22) and (2.24). These conditions also express the fact that there is no distributable surplus at M^* (condition 2.20). *There is therefore also equilibrium of the economy of markets.*

The first order condition implies that a same system of prices (x, a) exists, such that

$$A'_X = I'_A/I'_Y = a/x \tag{4.4}$$

There is local convexity at M^* for the consumption unit and local concavity for the production unit (§ 21.2). Condition (2.24*) reflects the fact that the curvature of the production function curve C is less than that of the indifference curve C^* tangent to it.

41.3 *Imputation and differentiation of tariffs.* – At M^*, for the production unit, we have

$$A''_{X^2} > 0 \qquad A'_X > A/X \qquad R = aA - xX < 0 \tag{4.5}$$

There is local concavity, increasing marginal return exceeding the average return, and *negative income if all the components of A and X are billed using marginal values x and a.*

Since *there is maximum efficiency, the global psychological value of production A is higher than the global cost xX* (§ 22.2). Otherwise, it would be possible to release a surplus by diminishing the quantity consumed A. This means that the budget of the production unit can be balanced by differentiating prices according to the quantities sold. *Such pricing is perfectly compatible with maximum efficiency.*

41.4 *Principle of maximum income and principle of distributable surplus.* – Income R is a minimum at the point M^* for given values of a and x under the condition of constraint $A = g(X)$, *and not a maximum, as suggested by the currently accepted principle of maximization of income, equilib-*

rium prices being taken as data. In other words, the application of the principle of maximum income to market equilibrium prices will result in departure from the situation of maximum efficiency M^*. This result, *which is paradoxical only in appearance,* stems from the fact that in reality, the production unit does not undertake exchanges with an indefinite entity but with the consumption unit considered (*condition of effective exchange*) (Table I, § 51). It follows immediately from the general theory of distributable surplus (§ 15), for at M^* the loss σ_u^* is effectively a minimum, and nil (§ 15.3 and 24), and any displacement compatible with the constraints from the point M^* departs from maximum efficiency and entails a loss.

Accordingly, if the production unit would seek to maximize its income, the prices a and x being considered as given, it would move away from the situation of maximum efficiency, *but such a maximization would make no sense since it could not actually materalize. Thus the principle of maximization of income for market prices considered as given has no economic meaning in this case. Only the search for an achievable surplus has a meaning, and in fact there is no potentially realizable surplus at M^*.*

42. *Some Basic Aspects of the Management of Transport Infrastructures*

Consider now a transport infrastructure which is assumed to yield increasing returns.

42.1 *The differentiation of tariffs.* – First, it is wrong to contend that the "unicity" or non-differentiation of tariffs (that is the use of the same price for every unit sold) which gives rise to the deficit is a necessary condition for maximum efficiency (§ 41 and 52; and Allais, 1967, § 83; 1981, p. 178-185 and 354-355).

Dupuit (1844) was the first to show, *in a masterly way*, that in certain cases, only a differentiated tariff can permit the effective rentability of a project when its *global utility* from the point of view of users is considered, and not its simple marginal use values, at which rentability may not be met.

It may be sometimes difficult to differentiate tariffs while preserving marginal condition (2.22), but this condition only applies for goods which can vary continuously, which is generally not true of transport activities. At worst differentiating tariffs may prevent a few users from using a transport infrastructure, but the loss will be of second, not first order magnitude (§ 24).

However, *condition* (2.22) *will not apply at all in many cases.* If, on a given day, a person takes the toll road from Paris to Lyon, his consumption is zero or one. As long as the toll remains below his psychological value of using the superhighway, its levy *does not affect* the conditions of maximum efficiency. It only engenders a transfer of income, which is neutral from the point of view of efficiency.

42.2 *The condition of budget equilibrium.* – In any event, because of the price distortions and losses which are generally caused by taxes (relation 2.26), using taxes to finance the deficit which would arise from the absence of differentiation of tariffs could generate other losses elsewhere. In addition, experience shows that by eliminating the budget equilibrium constraint, any subsidy generates inefficiency, while what really counts is that the production techniques used indeed be the best possible (§ 12.2 and 24). Lastly users should of course effectively pay for the costs attaching to the services they receive. *This is the generalization of the equimarginal principle for finite quantities.* Why should a journey made by travellers be financed by non-travellers?

42.3 *The decision to create a new transport infrastructure.* – A transport infrastructure, e.g., a motorway, is needed if the discounted present value of the *psychological values* (§ 22.2) of its future service exceeds the global expenditure for its construction plus the discounted present value of its future operating costs. The optimum size of the infrastructure is determined by the equality of the corresponding marginal values (§ 34.1).

42.4 *Pricing the services of an existing infrastructure.* – *If a forecast was perfect and the infrastructure accurately calculated,* it is always possible to find a system of tolls to finance operating and financial costs such that the conditions of maximum efficiency are respected. However, in any case the optimum toll system is always independent of past investment costs. *The general principle is that only the future counts*; the conditions of maximum efficiency at a given instant are *totally independent of past history.*

Should it turn out that the future was forecast inaccurately as happened often with the local railway systems built in France from 1890 to 1910, the optimum pricing of the corresponding infrastructures implies a toll which includes no financial charge (pure toll) as long as the traffic at that toll has not reached saturation point. This conclusion has a general value since the value of the service of a durable good corresponds to the equalization of supply and demand by the price system, *independently of any consideration of production costs* (§ 34).

Once an infrastructure is built, the only absolute necessity in all cases (irrespective of whether or not the initial calculations were correct and the forecasts accurate) *is to use it in the best manner possible* (that is, not jeopardizing maximum efficiency) and consequently not to charge a pure toll unless the demand at this toll exceeds the capacity of the infrastructure, in which case the toll system should be fixed at such level that demand equals capacity.

42.5 *Physical saturation and economic saturation.* – The existence of empty seats on a train might lead us to the conclusion that we are in a situation of increasing return, and that consequently the optimum tariff would be equal to the marginal cost, which is very low. Thus, to take the example of a passenger arriving at the Gare du Nord in Paris when there are still empty seats on the train to Calais, we might be led into thinking that since the marginal cost of carrying him is much less than the price of the ticket, it might be worthwhile granting him a reduction to induce him to take the train.

However the non-saturation of the train must be estimated economically. If traffic as a whole is dealt with (as it actually is, expressly or implicitly) so that the probability p of being unable to carry a passenger seated is fairly low (failure probability), it is certain that where $p = 1/1000$, for example, spare seats will be found on the train 999 times out of 1000. *Nevertheless the train must be regarded as economically saturated in such a situation*, and under this condition there is no reason to grant our passenger a reduction. The basic reason is that the service sold by railways is carriage *plus* the quasi-certainty of carriage, and in actual fact the tariff ensures equality between the anticipated demand and the capacity of the train with the failure probability reduced to p. *Economic saturation and physical saturation must therefore be carefully distinguished.* These observations can naturally be applied to all transport infrastructures.

42.6 *The consideration of marginal equivalences.* – Although the concept of marginal value is a particularly precious theoretical concept, since it simplifies many chains of reasoning and makes perfectly clear some of the basic results of economic analysis, *the calculation of marginal values in a given situation raises considerable difficulties.* To contend that to guarantee optimum management of railways, services need only be priced at their marginal cost *is candidly to state an apparently simple proposition, but one* *whose content is extremely complex and which in reality can only be mastered with great difficulty. There is generally an immense distance between the usual comments in the literature and concrete reality.*

The number of variables acting, e.g. in railway transport, is extremely high: distance, line profiles, mode of traction, types of locomotives, cars, wagons, rate of use of equipment, equilibrium or disequilibrium of the traffic, importance of peak hours, etc... The effects of all these variables must be combined. In fact and for instance there are at least tens of thousands of marginal values to be considered for French railways and their determination in practice yields at best an approximation. *The only effectively conceivable and in any case useful application of the estimates of the marginal values corresponding to a given situation is their use as indicators to determine the direction to be taken in altering operating conditions to move closer to optimum management.*

In reality, only by successive approximations based on the general theory of surpluses and the economy of markets is it actually possible to move closer to a situation of maximum efficiency.

42.7 *The external effects.* – If we have for instance to decide whether a motorway ought to be built or not, the question is obviously extremely complex, for what should be taken into account in the calculation is not only the present and future costs of building and maintaining the motorway, but also external effects, for example the nuisance for people living on the roadside caused by traffic noise, the gains made by those living near the exists and entrances, the consequent losses of traffic undergone by the existing railways, waterways and airports, the losses of clientele suffered by businesses situated near the former thoroughfares, the decrease in nuisance corresponding to the lower traffic on other thoroughfares, etc... *In so complex a case, only the theory of distributable surpluses and of losses, suitably applied, can bring about a correct economic calculation. But as a matter of fact this is only a generalization and an extension of the equimarginal principle to the most general discrete case.*

The conclusion that emerges from this analysis is that economic theory is as indispensable as it is difficult to apply.

V. – FOUR SIGNIFICANT MYTHOLOGIES IN THE CONTEMPORARY LITERATURE

Today there is a tendency to neglect the dynamic marginal approach based on the consideration of differences in marginal equivalences; and in the name of a so-called rigour it has been replaced by new theories. *A fortiori,* the general theory of surpluses which generalizes marginal analysis is simply ignored.

This development, which in reality, and despite the *too-widely* held belief to the contrary, represents *an immense step backward,* basically stems from the domination of the literature by four closely linked mythologies: – the mythology of the *market economy model*; – the mythology of the "unicity" of prices and the equalization of all supplies and demands by a same system of prices; – the mythology of general convexity; – and the mythology of abstract formalism.

These are only as many examples of the errors, strange aberrations, and more or less absurd fashions, flowing from the unquestioning acceptance of the "*established truths*" taught by the dominant "*establishments*", whose only real basis is their incessant repetition, which one finds only too often in the history of sciences.

In fact a short analysis of these mythologies can help to shed much light on the approach presented of the equimarginal principle, the general theory of surpluses and the economy of markets.

51. *The Mythology of the Market Economy Model*

The model of the market economy on which, following Walras, nearly all the contemporary theories are based is grounded on two essential, although usually insufficiently explicited assumptions: – *the existence at every moment of a single set of prices for the whole economy, whether equilibrium obtains or not*; – the transition from a non-equilibrium to an equilibrium situation through a system of exchange transactions (accompanied by the corresponding production decisions) *accomplished simultaneously, once the system of equilibrium prices has been determined.*

It is assumed that at every moment and in any non-equilibrium situation a certain set of prices is announced. These prices being considered as given, each consumption unit maximizes its preference index and formulates the corresponding demands and supplies; each production unit maximizes its net income and formulates the corresponding demands and supplies. All the supplies and demands thus formulated, corresponding to the set of prices announced, are compared *without a single exchange being realized.* The price of any good whose demand exceeds its supply is assumed to rise, and, conversely, the price of any good whose supply exceeds demand is assumed to drop. This produces a new set of prices. Then, still *without any exchange* *being realized*, new supplies and demands are formulated. A second set of prices is established, and the process continues *until* the announced set of prices actually equalizes the demand for and supply of each commodity. This is the Walrasian process of "*tâtonnement*".

For an initial situation characterized by a given distribution of resources among economic operators, the economic equilibrium of a market economy is essentially *defined by the three following conditions*: – the preference index of every consumption unit is maximum, market prices being taken as given; – the net income of each production unit is maximum, and positive or zero, market prices being taken as given; – for every good supply is equal to demand at the market price. In the equilibrium situation, the value of every index of preference is greater than or equal to its initial value.

In reality, the first two principles do not hold generally in situations of maximum efficiency, and they can be non valid (§ 41.4). And the third principle of the equalization of supply and demand by the price, i.e., the exchange of all units using the same price system, *is in no way a necessary condition for maximum efficiency*, and can be in contradiction to the search for maximum efficiency (§ 41.3, 42.1 and 52).

Since there is never a single set of prices for all operators in a situation which is not a state of equilibrium, the market economy model is totally unrealistic. Curiously enough, although, for instance, neither Edgeworth nor Pareto ever adopted this postulate as holding general value, the modern mathematical school places it as the foundation of its models.

Table I compares the fundamental principles underlying the market economy model and the markets economy model analyzed in Part I above. Although the two models define situations of maximum efficiency in the same way, their definition of equilibrium is *altogether different*.

The *essential difference* between the market economy model and the economy of markets model is that, in the latter, *the exchanges leading to equilibrium take place successively at different prices and that, at any given moment, the price sets used by different operators are not necessarily the same.* Whereas in the first model the final situation is determined totally by the initial situation, which correspondingly plays a privileged role without any real justification, in the second the final situation depends both on the initial situation and the *path taken* from it to the final situation (Fig. 6 and 7).

Whereas the market economy model postulates perfect competition and a large number, if not an infinity, of operators, the model of the economy of markets applies as well to the cases of monopoly as to the cases of competition.

Table 1.

MARKET ECONOMY AND MARKETS ECONOMY
FUNDAMENTAL PRINCIPLES UNDERLYING THE TWO MODELS
AND THEIR IMPLICATIONS

Market economy model	Economy of markets model
Assumptions	
(a) Transactions envisaged by operators are made in an *undefined* environment *in which partners are not specified.*	(a) Transactions are made between *specific* partners.
(b) The prices used by *all* operators are *the same,* whether general equilibrium obtains or not.	(b) The prices used are *specific* to the transactions.
(c) Supply and demand are *virtual* and do not result in actual exchanges as long as the condition that supply and demand be equal *for all goods* is not satisfied. The price of any good whose demand exceeds supply is assumed to rise, and conversely.	(c)) Supply and demand result in *actual* exchanges *if* a surplus is realized.
(d) The transition from the initial situation to the final one is made in one *single movement when and only when* the set of prices implementig the over-all equality of supply and demand for all goods has been found.	(d) Exchange transactions occur *even though* general equilibrium is not a consequence.
Definition of states of equilibrium	
Equilibrium exists if, under the set of prices considered, over-all demand and supply are equal for each good.	Equilibrium exists if there is no realizable surplus.
Definition of states of maximum efficiency	
Maximum efficiency exists if every preference index is maximum for given values of the other indices.	
Validity of the criteria for achieving a state of equilibrium	
– The two principles of – maximization of the preference index of a consumption unit at given prices – maximization of income of a production unit at given prices *have no general validity and can be shown to lead to wrong decisions.*	– The *dynamic* principle of realizing surpluses is of *absolutely general validity.*
– The principle of equalization of demand and supply by a same price (non-discrimination principle) *is in no way a necessary condition for maximum efficiency.*	
– The iterative price process which is considered as leading to a general equilibrium situation *may not be convergent and may never bring about a state of maximum efficiency.*	– The process of the gradual realization of possible surpluses leads to a state of maximum efficiency *in all cases.*

Not only is the market economy model unrealistic, but it also gives rise to considerable mathematical difficulties when an attempt is made to demonstrate the three fundamental theorems (§ 14.3 and 15.2). Whether differential calculus or set theory is used, these theorems can only be demonstrated under extremely restrictive conditions, and *the difficulties they imply are, from an economic standpoint, completely artificial, for they arise solely from the unrealistic nature of the model used. Paradoxically, whereas these restrictive assumptions are totally unrealistic, most of the theoretical difficulties encountered disappear once they are discarded* (Part I).

The market economy approach leads to imposing on any economic model, for it to be considered satisfactory, conditions which actually apply to a particular model, which are generally not fulfilled in reality, and for which, at all events, no rigorous justification can be found.

52. *The Mythology of the "Unicity" of Prices and of the Equalization of Demand and Supply through a Single Set of Prices*

It is true that where quantities vary continuously (in the mathematical sense), and functions can be differentiated, the system of marginal prices *in any situation of maximum efficiency* is the same for all operators (equimarginal principle). But *this condition is by no means a necessary feature of the dynamic evolution to a situation of maximum efficiency*. Actually it does not follow from the conditions of equilibrium and maximum efficiency that in moving from the initial situation to the equilibrium situation it would be necessary that all units should be sold or bought at the same price (unicity of prices).

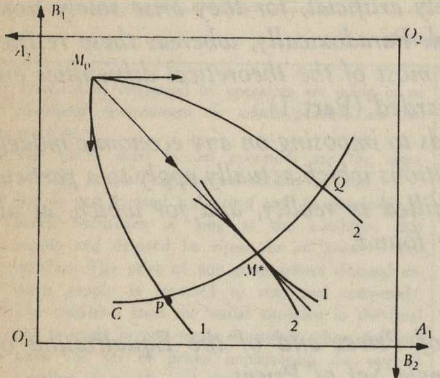
This can be clearly illustrated by the "*Pareto box*" *unduly referred* to as "Edgeworth's box", since Edgeworth makes no use of it in any of his works, whereas Pareto had made extensive use of it in Chapter III of his *Manual* (1909) (Jaffé, 1972; Allais, 1981, p. 67, n. 7). Pareto's box allows to represent all possible situations for two units performing exchange transactions, given the global quantities $A_1 + A_2 = A$ and $B_1 + B_2 = B$ of goods A and B that they initially hold (Figures 6, 7 and 8).

The curve C, the locus of the points of tangency of the indifference lines of the two consumption units, represents situations of maximum efficiency. In Figure 6, there is *a single exchange using the same price system* to switch over at once to the situation of maximum efficiency corresponding to the point M^* at which the common tangent of the indifference lines of the exchangers meets the point M_0. In Figure 7, the movement to the point of

TRANSITION TO AN EQUILIBRIUM SITUATION

I. GENERAL CONVEXITY

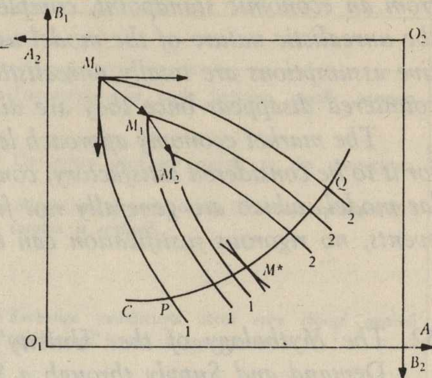
MARKET ECONOMY MODEL



Transition to the equilibrium situation through a single system of exchange transactions with a single system of prices

FIGURE 6

MARKETS ECONOMY MODEL

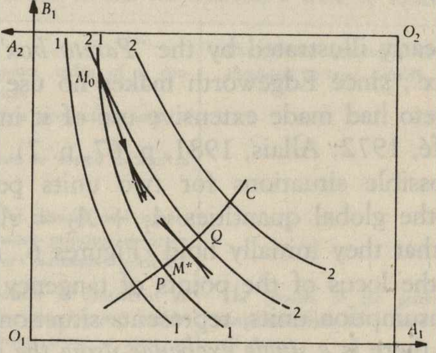


Transition to the equilibrium situation through successive exchange transactions with different systems of prices

FIGURE 7

II. CONVEXITY AND CONCAVITY

MARKETS ECONOMY MODEL



Transition to the equilibrium situation through successive exchange transactions with different systems of prices

FIGURE 8.

tangency of indifference lines is realized through *successive exchanges at different prices*. For each transaction, the quantities supplied and bought are indeed equal at the corresponding price, but for each good, starting from the initial situation M_0, the condition of equalization of supply and demand by the same system of prices, as in Figure 6, disappears. There is an infinity of possible final situations. *In both cases*, the preference index of each operator increases after each exchange, whether it is unique as in the first case, or includes several stages as in the second.

It is immediately verified that a path such that the displacement M_0M^* in Figure 6 is possible whatever the initial situation M_0 of the two units only if there is general convexity of their fields of choice (§ 21.2). This property remains valid in the most general case.

Figure 8 corresponds to Figure 7 for the case in which the fields of choice of consumption units 1 and 2 are respectively convex and concave. At M^* the index I_1 is maximum for a given I_2. The point M^* therefore corresponds simultaneously to a situation of maximum efficiency and a state of stable equilibrium of an economy of markets. It is possible to move from M_0 to a point such as M^* via a sequence of successive displacements in which the prices used differ and are advantageous for each consumption unit. But there is no system of prices which can be used to move *directly* from M_0 to a point M^* located between P and Q. At M^*, *the equilibrium of the economy of markets is stable, but the equilibrium of the market economy* is unstable. For the prices corresponding to M^*, the index I_1 is at a maximum subject to the budgetary constraint represented by the common tangent at this point, but the index I_2 is a *minimum* under the budgetary constraint.

In reality, the condition of "unicity" of prices is an ethical condition of equality of treatment which is not implied at all by the conditions (2.22) *and* (2.24) *of equilibrium and maximum efficiency*. It may even be incompatible with the achievement of a situation of maximum efficiency. This is for instance the case of Figure 8 and all production activities for which increasing returns obtain (Part IV).

It is only because of a completely arbitrary convention that the point M^* on Figure 6 can be considered as preferable to all the other points on the curve PQ. Certainly it is not unreasonable to consider that the application of this rule leads to a mutually acceptable compromise by the operators on the distribution of the surplus that can be obtained by moving from point M_0 to a state M^* of maximum efficiency, *but this is an ethical view which is in no way implied by the conditions of maximum efficiency, and which in any case* *cannot obtain if there is no general convexity.* Moreover, it assumes implicitly that the initial situation is ethically optimal.

In any event, from an ethical point of view, it is by no means evident that the use of a single system of prices, assuming that there is one, produces the most equitable results. It takes only very partial account of the difference between the market price and the psychological value for each operator, and treats those for whom the psychological rent is very high in the same way as those for whom it is very small or zero.

In his work "*Mathematical Psychics*" (1881), Edgeworth stated a theorem, relating to an economy in which there are no production operations, which provides an ethical justification for the principle of "unicity" of prices by introducing the possibility of *recontracting*. He considers an initial situation and a state of equilibrium in the Edgeworth sense, i.e., where one preference index is maximal for given values of the others. In this equilibrium, the preference indices I_1, I_2, ..., I_n of the operators will all be higher than in the initial situation. Then *any* group of operators is assumed to be given the possibility of "*recontracting*", i.e., of starting out from their original allocation of various goods and concluding a new set of contracts if, in this way, their preference indices can reach levels I'_1, I'_2, ..., I'_n such that $I'_i > I_i$ for any operator i of the group considered. *All exchanges are assumed to take place only when there is no further possibility of recontracting.*

If recontracting is possible, only some of the equilibria in the Edgeworth sense are feasible. Using intuitive reasoning, although not very rigorous, Edgeworth showed that the indeterminacy of equilibrium is reduced as the number of operators increases, and that, *if all fields of choice satisfy the postulate of general convexity*, the only possible equilibrium (the "*core*" of the economy) for an infinite number of operators is the market economy equilibrium corresponding to the initial situation considered and the use of *a single system of prices* (Edgeworth, 1881, p. 16-55 and especially 36-47; and 1891, p. 315-319. The Edgeworth Theorem has been demonstrated rigorously under the hypothesis of general convexity by Scarf, 1962; Debreu, 1963; and Scarf and Debreu, 1963).

The problem discussed by Edgeworth relates to competition for the appropriation of surpluses. Edgeworth's analysis shows that the possibility of recontracting generally yields a more equitable distribution of surpluses than would otherwise have occurred.

However, since Edgeworth's "recontracting" model pre-supposes that all exchanges are undertaken with the same set of prices, that no exchange occurs as long as any group of operators can "recontract" advantageously, and that general convexity is assumed, it is just as unrealistic as Walras'. The Walrasian "tâtonnement" process is merely replaced by Edgeworth's "recontracting", *but fundamentally the model remains the same: movement from the initial to the equilibrium situation in a single set of exchanges all at the same set of prices.*

53. *The Mythology of General Convexity*

Although the theory of general economic equilibrium *of a market economy* was developed without difficulty, once the first order conditions alone were examined (relations 2.22), major problems arose in the study of the conditions of stability and in the attempt to secure rigorous analysis for the second order conditions of the two theorems of equivalence (§ 14.3).

To do away with these mathematical difficulties, and not because of any analysis of the data derived from our introspection or from observation, many authors, such as Debreu (1959), *have assumed general convexity of the consumption units fields of choice and of the production units fields.*

As a matter of fact the two assumptions of general convexity of fields of choice and production actually play a key role in all demonstrations of the theories of equilibrium and maximum efficiency *using the market economy model and the set theory*. Debreu, for instance, notes that "*the convexity assumption is crucial because of its role in all the existing proofs of several fundamental economic theorems*". In fact, it has been impossible until now to demonstrate with the set theory that, in a market economy, a state of equilibrium is equivalent to a state of maximum efficiency, and vice versa, without postulating the general convexity of fields of choice and production.

However, the postulates of general convexity are formally contradicted both by introspective data and by empirical study of the whole production sector (§ 21.3).

Admittedly if the continuity and derivability hypotheses are accepted, it can be demonstrated, that in a situation of equilibrium, most operators are in a position of local convexity (§ 23), but from this it is impossible to conclude that *all* fields of choice and production *are convex everywhere. Economically the hypothesis of general convexity means that the marginal returns are never increasing. As a matter of fact this hypothesis implies absurd consequences*: either the optimum size of the production units is nil and there is an infinity of production units for each good produced, or the optimum size and the number of production units remains indeterminate.

It is, to say the least of it, astonishing to see how many contemporary works use preference functions and production functions, and how few are devoted to a study of their properties (on utility functions, see in particular

Stigler, 1950; Arrow, 1952, p. 529, and 1968, § 2; Georgescu Roegen, 1968; and Allais 1968 A, II, p. 109-137 and 1981, p. 309-317; on production functions see Allais, 1943, p. 181-211; 1968 A, II, p. 68-98; 1974 B, and 1981, p. 317-323; Arrow 1952, p. 530, and 1968, § 2; V.L. Smith, 1968; Walters, 1968).

54. *The Mythology of the Abstract Formalism of Set Theory*

Over the past thirty years, in economic research and even in teaching, there has been a steady trend toward the use of set theory and away from the differential calculus to demonstrate the three fundamental theorems (§ 14.3 and 15.2).

But as they rest on the market economy model and the hypothesis of general convexity, these theories do not correspond in any way to the observed facts and as a result they must be considered as absolutely illusory.

In reality, their formal simplicity is obtained at a prohibitive cost. Essential concepts such as marginal equivalence, surplus, and loss are no longer considered and have vanished totally from the analysis. Relationships such as those expressing the value of durable goods as a function of the value of their future services, those describing the first-order conditions of a state of equilibrium and of a state of maximum efficiency, and those describing the second-order conditions, are no longer made explicitly, despite their major importance in economics.

As for the accepted assumption of continuity, the new theories are just as approximate and unrealistic as the theories based on differential calculus, and they are actually incapable of analysing the question of increasing returns or the fundamental problem of indivisibility. By accepting the principle of general convexity, they lead to absurd results once applied to reality.

In any event, if we want to understand the real, what point is there in excluding the differential calculus at the same time as continuity is accepted? Admittedly, it has been known since Weierstrass that there are continuous functions for which no derivative exists for any value of the variable. But how could an economist with a sound mind give preeminence to a theory on the ground that it is more general because it includes such functions? It is really absolutely unreasonable to proscribe the use of differential calculus in the name of formal rigour, and so have to rely on set theory alone. In fact, from a mathematical point of view, the future of economic theory rests on the *joint* use of differential calculus and set theory, wherever these two sectors of mathematics can yield essential results as simply and as rapidly as possible.

Formal rigour is of little value if it is accompanied by a very serious distortion of the true nature of reality, and it is better to have an approximative theory that corresponds to actual reality than a formally rigorous theory that can only be built by seriously distorting the facts. The generality, simplicity, and rigour of the new theories of equilibrium and maximum efficiency are purely formal, for, from the standpoint of factual analysis, these theories are based on assumptions that are very restrictive, unrealistic, and wholly unacceptable. They deal rigorously with the problems they address, but these problems are *artificial*, and have nothing to do with the real world.

In fact, from an economic viewpoint, all the work based, for the past thirty years, on the market economy model and the postulate of general convexity has been built on sand. Devoid of any economic interest, it is no more than a fascinating intellectual plaything for mathematicians.

VI. – CONDITIONS OF A REALISTIC ANALYSIS OF THE DYNAMIC EVOLUTION OF THE ECONOMY IN REAL TERMS IN A FUTURE PROSPECT

61. *The Equimarginal Principle in Retrospect*

61.1 *The differences in marginal equivalences.* – From the beginning of the XVIII$^{\text{th}}$ century to marginal theory, economic analysis has progressively shown up the dynamic sequences of causality engendered by the differences in marginal equivalences as synthesized by relations (2.13), (2.16), and (2.16*).

This approach can be found with many authors who are not usually considered to be marginalists and nevertheless the marginal theory would not have been born without the guiding ideas they had set forth. Amongst them, the most noteworthy are: Turgot (1766-1769), Condillac (1776), Von Thünen (1842), and especially Ricardo (1817), Cournot (1838) and Dupuit (1844-1854).

61.2 *The equalities of marginal equivalences, first order conditions of equilibrium. The equimarginal principle.* – The founders of marginal theory, Gossen (1854), Jevons (1871), Menger (1871), and Walras (1874-1877), commented forcefully on equality of marginal equivalences (relations 2.22), as the outcome of causal sequences engendered by differences in marginal equivalences.

This analysis was later developed by Edgeworth (1881), Launhardt (1885), Auspitz and Lieben (1889), Wieser (1889), Marshall (1890), Irving Fisher (1892), Wicksteed (1894), Pareto (1896, 1901, 1906 and 1911), Wicksell (1901), Barone (1908). It was extended to the theory of capital by Boehm Bawerk (1884-1889), Irving Fisher (1906 and 1907), Allais (1943, 1947 A, 1962 and 1963 B).

61.3 *General conditions of equilibrium and maximum efficiency.* – Walras (1874) was the first to relate the analysis and discussion of the equimarginal principle to the conditions of general economic equilibrium, and after him, economic analysis focused mainly on: (*a*) the existence of an equilibrium of a market economy; (*b*) the convergence towards this equilibrium; (*c*) the stability of this equilibrium; (*d*) the two theorems of equivalence; (*e*) the concept of loss.

The most significant studies on these different points are as follows:

(*a*) Walras (1874-1877); Edgeworth (1881); Irving Fisher (1892); Pareto (1896-1897, 1906 and 1911); Hicks (1939 B); Allais (1943 and 1945); Arrow and Debreu (1954); Debreu (1959); Arrow and Hahn (1971).

(*b*) Walras (1874-1877); Allais (1943); Arrow and Hahn (1971).

(*c*) Hicks (1939 B); Allais (1943); Samuelson (1947); Arrow and Hahn (1971).

(*d*) Pareto (1896-1897, 1901, 1906, 1911 and 1916), and his immediate follower Barone (1908); Lange (1942); Allais (1943, 1945, 1947 A and 1952 A); Arrow (1951); Debreu (1959); Arrow and Hahn (1971).

Actually, the substance of the theorems of equivalence had already been grasped more or less clearly by Condillac (1776), Adam Smith (1784), J.B. Say (1819), Walras (1874-1877), Auspitz and Lieben (1889); Irving Fisher (1892). The interpretation of these theorems has been critized by Launhardt (1885) and Wicksell (1901).

(*e*) Cournot (1838); Dupuit (1844, 1849, 1853, 1854); Hotelling (1938 and 1939); Frisch (1939); Allais (1943); Debreu (1951); Boiteux (1951).

Pareto's proof of the theorems of equivalence takes only first order conditions into account; it confuses average and marginal costs; it does not draw the necessary distinction between the differentiated and the non-differentiated sectors; it does not take time and consequently interest into account; and does not provide the exact conditions of validity of the theorems of equivalence. Lange's proof, founded on the theory of restricted maxima and the consideration of Lagrange multipliers, is far more satisfactory than Pareto's, but it takes only first order conditions into account, and not the decomposition of production as a whole into distinct industries, and of each industry into distinct production units. In addition, it does not take time into consideration.

The most systematic and complete analysis of the market economy model *using differential calculus* is to be found in Allais (1943, 1945 and 1947 A). This analysis is free of the hypothesis of general convexity in the production sector; it takes account of second order conditions, and of the distinction between the differentiated and non-differentiated sectors; it takes account of time, of the future preference functions of present as well as future generations; and it sheds full light on the arbitrary nature of the distribution of income, which, in general, had been relatively neglected in earlier studies. Condition (2.20) is used to yield two proofs of the theorems of equivalence. The first is based on the consideration of the coefficients of equivalence, the second on the consideration of the Lagrange multipliers (1943, p. 604-637). A proof is given of the existence of an equilibrium, the stability of this equilibrium and the convergence of a market economy towards this equilibrium (1943, p. 486-493). This proof is given only under very restrictive conditions, but these conditions are plausible, and the proof has the merit of posing question (*a*), (*b*) and (*c*) very clearly for the first time in the literature. It is based on the consideration of a characteristic function, the sum of the absolute values of the differences between the values of supply and demand for the different goods. It results from the assumed hypotheses that this function tends towards zero as the market economy evolves. After a great many applied studies, Allais was finally led, in 1967, to replace the market economy model by the model of the economy of markets.

The most significant study of the market economy model using set theory is that of Debreu (1959). His proof of the theorems of equivalence is free of the hypothesis of differentiability, but pre-supposes the continuity and general convexity of all fields of choice and production.

Some sketches of the theory of an economy of markets were caught by Edgeworth (1881), Pareto (1906) and Allais (1943). A systematic development of this theory was presented by Allais (1967, 1968 A, 1968 B, 1971, 1974 A, 1981 and 1985).

On marginal equivalences and related issues see: Moret (1915); Hicks (1939 A); Samuelson (1947); Hutchison (1953); Schumpeter (1954); Arrow (1968); Mishan (1968); Blaug (1970-1985); Arrow and Hahn

(1971); Collison Black, Coats and Goodwin (1973); Dehem (1984); and Allais (1952 C, 1968 C, 1968 D, 1971, 1973 A, 1973 B, 1974 A and 1981).

On the management of production units with increasing returns, see: Dupuit (1844, 1849, 1853, 1854); Hotelling (1938 and 1939); Frisch (1939); Alchian (1968); Meyer (1968); Vickrey (1968); Blaug (1985, p. 601-613); Allais (1947 B, 1963 A, 1964, 1965 A, 1965 B, 1981, p. 159-198).

On the use of mathematics and the contemporary formalism, see in particular: Debreu (1959 and 1985); Hutchison (1977, p. 62-97 and 161-170); Woo (1985), and Allais (1943, p. 24-40, 1947 A, p. 527-535, 1949, 1952 C, p. 42-57, 1954, 1968 F, 1971, 1974 A, 1979 B and 1984, p. 63-71).

61.4. *General characteristics of the development of economic thought.* – On the whole, the evolution of the literature since the XVIIIth century can be described by a few basic features:

– The very realistic *dynamic* approach of equivalence differences in the discrete or the continuous case was progressively replaced by the purely *static* analysis of the equality of marginal equivalences in a situation of equilibrium and maximum efficiency.

– Theoretical analysis concentrated growingly in the market economy model and the hypothesis of general convexity, most markedly in the current mathematical literature, whereas in concrete reality, this model and its assumptions are nothing other than *extravagant.*

– The literature was continually influenced, implicitly or explicitly, by normative conceptions identifying more or less a state of maximum efficiency to an ethically optimum one. A good illustration is provided by the ill-chosen Anglo-Saxon term "*optimum allocation of resources*" used to describe the equilibrium state of a market economy corresponding to a given initial situation, whereas the initial situation cannot in general be thought of as ethically optimal – or, for that matter, the path leading from it to the associated state of equilibrium.

– Much of the contemporary literature on equilibrium theories went progressively under the control of pure mathematicians who are more concerned with mathematical theorems than with analysis of the real world.

– Finally, whereas the founders of the marginal approach and their immediate successors based all their analysis on the consideration of cardinal preference indices (cardinal utility) and its decreasing as a function of the quantities consumed (corresponding to situations of local convexity), this approach has gradually been set aside from Pareto's time (1906), and in the upshot, virtually proscribed, in fact completely unduly.

Lack of space precludes an analysis here of this (powerfully suggestive) historical development and of the causes, which have determinated it, but it is probably correct to draw at least one teaching from this evolution: the "truths" of any period must constantly be subjected to merciless critical analysis, with unceasingly reminding Pareto's statement: "*The history of science reduces to the history of the errors of competent men*". Such an analysis would inevitably lead us today to revise the conceptions that rule current thinking.

62. *The Foundations of a Reformulation of Economic Analysis in Real Terms*

From the foregoing a double conclusion emerges: – despite all its limitations and difficulties of application, *the classical theory of marginal equivalences is irreplaceable to make understandable the underlying nature of all economic phenomena; – the general theory of surplus* (Part I), *of which classical marginal theory is only a special case* (Part II), *allows to extend the propositions of marginal analysis to the most general case of discrete variations and indivisibilities.*

As important as the analysis of the conditions of general equilibrium and of maximum efficiency may be, the analysis of the dynamic processes which enable surpluses to be generated from a given situation, is *much more important.* From this point of view the analyses by Dupuit, Jevons, Edgeworth, Pareto, and the marginal school and its predecessors in general, *appear much more realistic than the contributions which rest only upon the consideration of Walras' general model of equilibrium.*

In fact, what is really important, is not so much the knowledge of the properties of a state of maximum efficiency as the rules of the game which have to be applied for the economy effectively to move nearer to a state of maximum efficiency.

The *decentralized search for surpluses* is truly the dynamic principle from which a thorough and yet very simple conception of the operation of the whole economy can be derived. *Whereas in the market economy model the search for efficiency is essentially focused on the determination of a certain set of prices, the analysis of the economy of markets model is based on the search for potential surpluses and their realization.* Not only is the economy of markets model much more realistic than the market economy model while lending itself to much simpler demonstrations, but also these demonstrations *are not subordinated to any restrictive assumptions about continuity, differentiability of functions, or convexity* (Parts I and V). All of economic dynamics is brought back to *a single principle*: the minimization of loss for the economy as a whole.

In fact, any criticism that might be made of the theory of the markets economy also applies to the other theories, but the opposite is not true. In all instances where the assumptions of continuity and differentiability can be made, at least as first approximations, the markets economy theory makes it possible to derive the essential propositions of classical theory on: marginal rates of substitution; the existence of a unique set of prices at equilibrium; local convexity conditions for most operators; the principles, for corresponding cases and for a state of equilibrium, of maximizing the preference indices of consumption units and the income of production units, market prices being taken as given; the principles of management of sectors operating under increasing returns; etc.; but it shows their real meaning and the limits within which they are valid. It shows in particular that the single price system condition is valid only at the margin and that, contrary to the prevailing view, *maximum efficiency does not require that all units of one good be exchanged at the same price, nor that the prices used equate over-all supply and demand.*

63. *Model-building and the Real World: The Major Criterion of Experience*

Any model is necessarily an abstraction of reality. Obviously, no theory can represent all reality. But while abstraction is a necessity, the way it is done is not a matter of indifference. *Reality can be simplified to advantage and without danger if the simplification is such that it does not change the real nature of phenomena. On the other hand, under no circumstances should the search for simplification lead to a change in the nature of reality.*

The basic rule should be subordination to the data of observation and experience. Assumptions should not be chosen as a function of a theory; it is the theory that should be chosen so that its assumptions are in line with observed data. A theory should be selected not because of the aesthetic qualities of its mathematics and of its theorems, but as a function of a *single criterion,* the analysis of facts and the conformity to facts.

Any model that does not conform to reality either in its assumptions or in its results must be rejected as inappropriate, *however* aesthetically satisfying the mathematical reasoning used in the phase of logical deduction. *For the economist, as for the physicist, the purpose is not to use mathematics as an* *end in itself, but primarily to grasp reality*, and never to dissociate economic theory from its applications.

The use of fairly complex mathematics is not a drawback in itself, *but it is only justified if study of the real world makes it unavoidable*. In all other cases, the simpler analytic procedures are the best. The economist has no interest in deploying a very abstract mathematical construct if the additional information thus provided is of illusory value, and in no case is there any justification for studying unreal problems. A good mathematician cannot be a good economist because he is good at mathematics. Depending on the case, mathematics in economics are like Aesop's tongue, the best and the worst of things.

Today, complex theories are built on the model of the market economy and the postulate of general convexity, *without their authors worrying about what relations the market economy model has, if any, with concrete reality and what fields of choice and production sets really are. Paradoxically, from the scientific point of view, infinitely more care has been lavished on the mathematical elaboration of models than on the discussion of their structure and assumptions as they relate to the analysis of facts.* An immense flood of mathematics has relegated the discussion of essential assumptions to the background, and too many authors seem to be more preoccupied with the enunciation of theorems in pure mathematics than with analyzing real facts. Nevertheless, as Pareto commented: "Even from an exclusively theoretical standpoint the use of mathematics adds nothing to the rigour of proofs. *If the premises are false, mathematically drawn deductions will be just as erroneous as those provided by ordinary logic*".

COMPLEMENTS

For a thorough analysis of the foregoing, see:

I. *On the general theory of surpluses and the economy of markets in the general case; and on the fundamental theorems*:

Allais (1943, p. 112-177, 181-211, 604-656; 1967, § 8-65; 1968 A, vol. II; 1968 B; 1971; 1974 A; 1981, p. 27-48; 1985).

II. *On the theoretical foundations of the equimarginal principle*:

Allais (1943, p. 604-656; 1945; 1952 C, p. 28-32; 1967; 1968 A, vol. II; 1971; 1973 A; 1973 B; 1974 A; 1974 B; 1981). – *Illustrative models*: (1943, Annexe I, p. 4-24; 1945, p. 57-69). – *Applications to international trade*: (1959, p. 283-294; 1961; 1970, p. 87-100).

III. *On the marginal equivalences and the approach towards situations of equilibrium and maximum efficiency; limits of the equimarginal principle; extensions to the cases of perfect and imperfect foresight*:

Allais (1943, p. 343-384; 1947 A, p. 23-228; 1964; 1967; 1968 A, II). – *Illustrative models*: (1947 A, p. 631-771). – *Capitalistic optimum theory*: (1947 A, p. 179-228; 1962, 1963 B; 1968 E). – *Demographic optimum theory*: (1943, p. 749-785). – *Case of Risk*: (1952 A; 1952 B; 1979 A).

IV. *On the management of production units with increasing returns and the applications of the marginal cost theory*:

Allais (1943, p. 635-637, 678-679; 1947 B; 1951; 1953; 1963 A; 1964; 1965 A; 1965 B; 1967, § 83; 1968 A, p. 248-249; 1981, p. 178-185).

V. *On the mythologies of the literature*:

Allais (1967, § 74, 75, 76, 83; 1971; 1974 A).

VI. *On a realistic approach of the economics dynamics in real terms*:

Allais (1943, p. 24-40; 1954; 1967; 1968 B; 1968 F; 1971; 1974 A; 1981; 1985).

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IL PRINCIPIO DI UGUAGLIANZA DELLE EQUIVALENZE MARGINALI. SIGNIFICATO, LIMITI E GENERALIZZAZIONE

Questo saggio si propone di analizzare il "principio di uguaglianza delle

equivalenze marginali" relativamente al *suo significato, ai suoi limiti e alle sue generalizzazioni.*

La *Parte I* presenta innanzitutto una breve rassegna dei concetti e proposizioni economiche fondamentali *nel caso più generale* dove non possono essere ammesse le ipotesi di continuità e di differenziabilità che il principio di uguaglianza delle equivalenze marginali presuppone. Questa rassegna si basa sulla considerazione di un modello *molto generale* di continuità, di differenziabilità o di convessità che non comporta alcuna condizione limitativa, cioè il modello dell'economia di mercati (contrapposto a quello dell'economia di mercato) che l'autore aveva proposto nel 1967, e di una teoria, *la teoria generale dei surplus* che egli aveva elaborato dal 1946 al 1967 e che generalizza la teoria marginalistica classica.

Nell'ipotesi di continuità e di differenziabilità, ma senza alcuna ipotesi di convessità generale, la *Parte II* presenta un'analisi del principio dell'uguaglianza delle equivalenze marginali considerato come il risultato dell'evoluzione dinamica delle differenze delle equivalenze marginali e al tempo stesso come una condizione di equilibrio stabile e di efficienza massima. Le applicazioni e le generalizzazioni di questo principio dominano tutta l'analisi dell'economia in termini reali.

La *Parte III* mostra i significati e limiti dell'applicazione dell'approccio marginalistico (e quindi, in particolare, della teoria del costo marginale), soprattutto per quanto riguarda le indivisibilità e l'imprevedibilità delle situazioni future.

La *Parte IV* illustra l'analisi marginalistica nel caso delle unità di produzione a rendimento crescente e dell'economia delle infrastrutture dei trasporti. In particolare mostra la netta superiorità nei casi considerati del modello dell'economia di mercati su quello dell'economia di mercato.

La *Parte V* presenta un'analisi critica di quattro mitologie che dominano la letturatura contemporanea e che si basano su una quantità di ipotesi assurde e di teoremi che non corrispondono ad alcun problema reale: la mitologia del modello dell'economia di mercato, la mitologia dell'unicità dei prezzi e dell'ugualizzazione delle domande e delle offerte per mezzo di uno stesso sistema di prezzi, la mitologia del postulato di convessità generale, e infine la mitologia del formalismo astratto della teoria degli insiemi.

La *Parte VI,* infine, presenta una breve panoramica dell'evoluzione storica e specifica le condizioni di un'analisi realistica dell'evoluzione dinamica dell'economia in termini reali in una prospettiva futura. La *ricerca decentralizzata delle eccedenze* appare come il principio dinamico dal quale si può far derivare una concezione elaborata e al tempo stesso fondamentalmente molto semplice del funzionamento di tutta l'economia che permette di generalizzare l'approccio marginalistico nel caso più generale.

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TINBERGEN'S CONTRIBUTION TO SOCIAL ECONOMICS

by
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1. – Jan Tinbergen shared with Ragnar Frisch the first Alfred Nobel Memorial Prize in Economic Science in 1969. Tinbergen has done pioneering work in econometric model building. Indeed the Nobel citation commended him for having developed and applied dynamic models for the analysis of economic processes. At the same time it should be noted that his interest in economics extends far beyond issues of methodology and technique. A Festschrift in his honor edited by Sellekaerts (1974a, 1974b, 1974c) covers three volumes and includes essays covering the fields of econometrics, economic theory, international trade and finance as well as economic development and planning. Bent Hansen (1969, p. 336) has described him as "a truly Schumpeterian figure in economics. A man who has triggered off developments in many different directions". His writings reflect not merely superior technique but also an acute concern with social equity as a determinant of human welfare.

2. – Tinbergen has written dozens of articles and several books in the field of development planning alone. He has written not merely on problems of individual countries (industrialized as well as developing) and of groups of countries but also on the prospects of (and difficulties in the way of) achieving an international economic and social order. The recognition accorded to his accomplishments in mainstream economics has not extended to his equally path-breaking contributions in the sphere of social economics. The object of this paper is to offer a preliminary survey of these latter contributions. For this purpose we divide Tinbergen's works into three categories: (a) development planning; (b) global economy; and (c) optimal social order. We do not pretend to offer a comprehensive or a definitive

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survey of Tinbergen's contributions in the area of social economics. That would probably require at least a long monograph. We also exclude methodological and technical issues discussed at length by him in his works which would be included in the above-mentioned three categories. Our focus is on social policy aspects of Tinbergen's contributions. To those who are familiar with his works it should come as no surprise that his interest in economics was matched by (and probably came from) a deeper concern with humanistic issues. Bent Hansen (1969, p. 336) perhaps expressed this most eloquently: "... the humanitarian idealism which Nobel wanted to reward has no representative in our profession so fine and noble as Jan Tinbergen. As much as he is respected in the privileged world, he is beloved by the underprivileged, the underdogs. Always at their service, always at their side, always working on improving their conditions, Tinbergen would be an equally worthy candidate for the Nobel Peace Prize".

3. – Tinbergen's publications dealing with development planning first appeared after World War II when he became Director of the Central Planning Bureau of the Netherlands. His early writing in this area dealt with issues of development in the Netherlands. Among his first publications discussing problems of less developed countries were his comments on the First Five Year Plan of India. Before World War II much of the discussion on the problems of less developed countries was descriptive in nature. Issues of economic development and development planning were not then recognized as a separate field of enquiry. This is illustrated by the fact that Schumpeter's contributions in this area were at the time considered as belonging to business cycle theory. Problems of development were viewed within a more analytical (as opposed to descriptive) framework perhaps for the first time by Rosenstein-Rodan (1943) in his famous article on industrialization in Eastern and Southeastern Europe. Rosenstein-Rodan's analysis was mainly qualitative in nature. With the development of work on programming and activity analysis, problems of development began to be formulated and discussed in more quantitative terms in the late 1940s and the early 1950s.

4. – Tinbergen was among the very first economists who sought to bring about a synthesis of the qualitative and the quantitative points of view. In his earlier works, he was preoccupied with the analytical issues and perhaps even fascinated by the challenge of the technical problems. But even in the early writings on the subject, it is clear that his goal was to be able to apply the analytical tools to the problems of development. And he

always viewed problems of development not as abstract phenomena but as issues having very significant implications for vast numbers of human beings. His work, *The Design of Development* (1958), served as a basis for discussions relating to development policies and problems of programming and project appraisal at the World Bank. His advocacy of planning was not always well-received by those favoring free markets. His views were not appreciated perhaps because their implications were not fully understood. He stated explicitly and forcefully that one of the four principal objectives of development planning should be "to take measures designed to facilitate and to stimulate private activity and investment" (1958, p. 4). At the same time government's role in providing "a minimum of social security" and "to correct the most extreme inequalities in income – inequalities easily conducive to social unrest and lack of cooperative spirit in production" was emphasized. He believes that one of the criteria of optimal planning is the degree of democracy allowed in the sense of satisfying justified desires of interested groups to have an opportunity of giving an opinion, or even participating in a number of decisions about the plan (Tinbergen, 1964). His works show a bias in favor of planning, but not in favor of Soviet-style planning from above; for, his works also show a bias in favor of democracy. "If democracy is to be saved, all interests involved must be represented".

5. – Tinbergen recognized the role of non-economic variables in planning and more generally in economic policy. Thus, he states: "Economic policy ... has to reckon with many aspects originating from very different realms of life ...: institutional, juridical, technical ... and psychological ... A sound policy has to satisfy certain principles set by or accepted by the majority of parliament, and to express some aspects of what the people consider to be the basis of the nation's public life. A number of such principles have been laid down in the constitution ... Some of them may be unwritten rules of social decency, or vaguer: principles such as some notion of social equilibrium" (Tinbergen, 1970a, p. 74, 75).

6. – Tinbergen was an advisor to the League of Nations and, after World War II, to the United Nations. He wrote extensively on globlal problems. While he brought his technical economic expertise to bear in these discussions, his writing was simple. For example, in *Lessons From the Past* (1963, p. VII): "an attempt is made to summarize, in simple language, the main lessons which the past half century has to teach us with regard to our economic and social regime". While acknowledging that "The pace of human life... has been very intense. The human race has experienced violent

ups and downs of fortune ... The world has become smaller and smaller, and yet the extremes of prosperity and poverty are wider than ever. There is an increasing tendency to make daily life safer, but never have we been exposed to graver risks than those which face the world ..." (1963, p. 3), he does not shy away from complex problems (political, social, economic) of the West, the East and the South. He discusses peace prospects as well as issues of social justice within a nation and between nations.

7. – He identified four major problems (having some degree of interrelationship) facing the World in the late 1950s and early 1960s: (i) spread of communism and potential conflict with the West; (ii) transition problems arising from the end of colonialism and the economic, political as well as social problems facing the former colonies; (iii) rapid increases in population in less developed countries; and (iv) dangers posed by the threat of nuclear warfare. Tinbergen criticized the centrally planned or regimented economies for their inefficiencies; but he also criticized the pure free-enterprise economy as being sub-optimal. He felt that "mixed" systems were better both from the point of view of (i) efficiency (i.e., best means of reaching clearly defined goals) which requires systematic planning, partly at a world level, and of (ii) solidarity (i.e., regard for the well-being of groups, as contrasted to individual members; an element of common responsibility) which requires international coordination or integration. He found the political polarization (into two groups: the West vs. the East) of countries disequilibrating. He felt that the uncommitted developing countries could play a crucial role in the interest of world peace. This would however require that such countries should not be induced or forced to join either the Western or the Eastern bloc. He outlined (1962) essential elements which would form the basis of a "new international order". These elements included suggestions about political, economic and social issues. He was critical of the relatively small part of the intellectual resources devoted to the problems posed by international conflicts as compared to those directed at solving scientific problems.

8. – Tinbergen carried his vision of an international order further when he presented a paper at a symposium in 1969 (1970b). In the paper he discussed what he called "the disastrous consequences of nationalism and racism the world has been facing for centuries". Therefore, he argued that a necessary condition for survival is to overcome some forms of nationalism. He proposed the concept of a "true global economy" not as a realistic concept for the near future but a frame of reference for present action. His

conditions for a global economy were based on "a good deal of intuition" which "comes in wherever the explicit solution of a scientific problem has not yet been found". These conditions included (i) price uniformity (ii) optimum division of labor (iii) an international redistribution system (iv) redirection of education and research efforts and (v) changes in institutional structures. He concluded that "many tasks can and should be left to low levels of decision making, including national levels. For a few key decisions, however, the national level of decision-making is non-optimal and in a number of cases even disastrous" (1970b, p. 344).

9. – Even when Tinbergen was introducing techniques which make economic planning possible and when he was promoting their application to economic development, he was reminding economists as well as policy-makers that human welfare (well-being) and equity must be considered to be the goals of all economic activity. He included discussion of the optimum social order under the heading of what he called the qualitative aspect of a development plan. "For any development plan, but more especially for a long-term plan, one fundamental question has to be asked: is the social order, in other words, the complex of institutions of which the society is made up, the best? In many cases, it will be found that this social order ... is no longer suited to meet the demands of a modern society and modern development ... Consequently, a really well-thought-out development plan must also clearly envisage the form of the new social order aimed at ... By 'social order' is meant the whole complex of institutions with their rules and regulations which form the society and within which individuals are able to pursue and develop their activities. ... Every complex of institutions of this kind and the rules which govern their conduct result in a certain pattern of human behavior in both the material and the spiritual spheres. This in turn will determine production, distribution and consumption and ultimately human prosperity and its equal or unequal distribution over the whole population. The all-embracing and at the same time extremely difficult problem of finding the best social order is in principle dependent on whether the pattern of society which will lead to the greatest welfare of all is found from among the many different social orders that are in fact possible" (Tinbergen, 1967, p. 64, 65).

10. Tinbergen believes that the present international institutions like the International Monetary Fund, the World Health Organization and the United Nations Security Council are vested with "too little competence and too little power" (1985, p. xi). There is a lopsided distribution of power in

favor of national authorities and a near-vacuum at the world level because of nationalistic feelings and national sovereignty. The nationalistic feelings are connected with and often reinforced by languages spoken, customs and habits. The situation has become dangerous because the awareness of one's national heritage is coupled with the militarization of public life and the rapid progress of armament technology. A comparison of the main features of an optimum world social order with the existing world order leads him to the conclusion that it is "shocking and commensurate with the fears of many for the future" and that it is not enough to have local or even national legislation and action to deal with global issues like use of environmental resources and controls.

11. – Tinbergen referred frequently to rapidly growing population in some parts of the world and to uneven distribution and use of environmental resources as problems within his scheme of optimal world social order. However, he has not discussed any specific solutions to these problems. There is ample evidence that the highly unequal patterns of economic and social progress both among individual countries and geographical regions as well as disparities within nations disturb him. The adverse impact of unequal rates of growth of population on the inequalities of distribution of incomes (intra-national as well as international) is also a very negative feature for him. Large segments of world population, in his view, must feel a rejection of their basic human needs and rights including the right to participate actively in the process of decision-making. Tinbergen's writings on the subject of optimal world social order in the face of these existing unpleasant realities reflect his call for the "need for greater world-wide rationality" (Pajestka, 1973).

12. – Tinbergen's contribution to social economics lies in having brought the big global issues of optimal social order to the forefront. He brought to this discussion a clearer description of the aims of social and economic policy and of the means of achieving these aims. He viewed all the measures proposed in social and economic policy as a single whole resulting from the close connection that exists between economic and social factors. He has constantly sought to bring a maximum degree of objectivity in discussions involving economic, social and political issues. For example: "Social scientists are asking themselves continually ... whether their own system is the best or not, and if not, what changes do they propose. Opinions on this question diverge widely and a considerable number of politicians have rather doctrinaire views. Some schools of thought have expressed

their opinion, or rather belief, in such a way that they contribute to an increasing polarization: the creation of two opposite camps, usually with a stamp on them, such as socialism or capitalism. Both as a citizen and as a social scientist I dislike this tendency, and I think it is an increasing danger ... I am in favor of depolarization, and science generally has the task to draw questions out of the sphere of emotions and shift part of our answer to the realm of objective observation and reasoning. Part only, since we know that not all questions are susceptible to objective treatment. Wherever we succeed in shifting the frontier between belief and science, we have made progress" (Tinbergen, 1972, p. 23-4). Perhaps Tinbergen's contribution to social economics is best summarized in the following remarks he made in 1972: "Welfare economics is the chapter of economics dealing with the question concerning what condition must be fulfilled in order that social welfare be a maximum, subject to the restrictions within which human society has to live. The definition of social welfare is correctly supposed to be given to the economist and to contain at least one basic ethical element ... to my taste, several economists have formulated in too narrow a way what are the unknowns of the central problem of welfare economics ... the unknowns are a number of economic variables or entities and that is all. In my opinion the problem is much deeper, and the real unknowns of it are the set of institutions (or the several alternative sets of institutions) which by their operation will bring us that optimal situation or rather, optimal development over time ... While it has been customary for welfare economists to ignore the ethical factors as long as possible and only to consider it after the economic analysis has been finished, I prefer to discuss the social welfare function in the beginning, implying that an ethical choice is made at the beginning. This enormously simplifies the ensuing economic analysis ... " (Tinbergen et al., 1972, p. 26-7). Is that not what social economics is all about?

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IL CONTRIBUTO DI TINBERGEN ALL'ECONOMIA SOCIALE

Jan Tinbergen nel 1969 divise con Ragnar Frisch il primo premio Nobel per l'economia. Tinbergen è stato un pioniere nella costruzione di modelli econometrici. La motivazione del premio Nobel era l'aver sviluppato e applicato modelli dinamici all'analisi dei processi economici. Al tempo stesso si deve ricordare che il suo interesse per l'economia si estende molto oltre a questioni metodologiche e tecniche. Un Festschrift in suo onore pubblicato da W. Sellekaerts in tre volumi include saggi di econometria, teoria economica, commercio e finanza internazionale, sviluppo economico e pianificazione. Bent Hansen l'ha descritto come « figura veramente schumpeteriana nell'economia. Un Uomo che ha suscitato sviluppi in molte diverse direzioni ». I suoi scritti riflettono non soltanto una tecnica superio-

re, ma anche una profonda attenzione per l'equità sociale come determinante del benessere umano.

Soltanto sulla pianificazione dello sviluppo Tinbergen ha scritto dozzine di articoli e parecchi libri. Egli ha scritto non soltanto sui problemi di paesi singoli (sia industrializzati che in via di sviluppo) e di gruppi di paesi, ma anche sulle prospettive (e relative difficoltà) relative al raggiungimento di un ordine economico e sociale internazionale. Il riconoscimento che è stato dato alle sue opere di economia non è stato esteso ai suoi contributi ugualmente pionieristici nel campo dell'economia sociale. Scopo di questo articolo è di offrire una rassegna preliminare di questi contributi. Dividiamo le opere di Tinbergen in tre categorie: *a*) pianificazione dello sviluppo; *b*) economia globale; e *c*) ordine sociale ottimo. Non pretendiamo di offrire una rassegna onnicomprensiva né definitiva dei contributi di Tinbergen nel campo dell'economia sociale perché questo richiederebbe probabilmente per lo meno una lunga monografia. Escludiamo anche i problemi metodologici e tecnici da lui trattati in modo esteso nelle sue opere che sarebbero inclusi nelle tre categorie sopra menzionate. Il nostro principale interesse è sugli aspetti di politica sociale dei contributi di Tinbergen. Coloro che conoscono i suoi scritti non dovrebbero essere sorpresi nel costatare che il suo interesse all'economia si accompagnava a (e forse derivava da) un più profondo interesse per i problemi umani.

Tinbergen è stato fra i primissimi economisti che cercarono una sintesi fra il punto di vista qualitativo e quantitativo. Nei suoi primi lavori egli si dedicò a questioni analitiche e forse era anche attratto dalla sfida dei problemi tecnici. Ma anche nei primi scritti su questi argomenti, è evidente che il suo scopo era di riuscire ad applicare gli strumenti analitici ai problemi dello sviluppo. Ed egli ha sempre considerato i problemi dello sviluppo non come fenomeni astratti, ma come problemi che avevano implicazioni importantissime per un vasto numero di esseri umani.

Il contributo di Tinbergen all'economia sociale consiste nell'avere portato alla ribalta i grandi problemi globali dell'ordine sociale ottimo. Egli discusse, descrivendoli, gli scopi della politica sociale ed economica e i mezzi di raggiungere questi scopi. Considerava tutte le misure proposte in politica sociale ed economica come un unico insieme che risulta dalla stretta connessione fra fattori economici e sociali. Egli ha costantemente cercato di portare il massimo di obiettività nelle discussioni sui problemi economici, sociali e politici.

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ON ENDOGENOUS OIL MARKET FLUCTUATIONS

by
LORENZO RAMPA *

1. *Introduction*

The striking oil price increase during the seventies and the early eighties is still a matter of alternative theoretical explanations. Furthermore, while discussions are going on, the recent and equally astounding price fall suggests that these tumultuous changes may have the character of disequilibrium fluctuations rather than of movements from an equilibrium to another. In fact, it seems that the oil market has resumed its irregular behaviour after the period of strict control by the Seven Sisters.

The purpose of the present paper is to look for the conditions of endogenous oil price oscillations by means of a stylised dynamic model. Obviously this does not mean that one should exclude any role for exogenous shocks, like wars or embargoes, but rather that one should adopt a disequilibrium approach to the problem.

As a touchstone one could think of the popular non collusive theory of oil pricing based on the target revenue hypothesis. It is well known that the latter implies a backward bending supply curve, at least for prices above those needed to support the target; and that this unusual supply shape may generate multiple equilibria (Teece, 1982).

Thus the price increase could have been caused both by a demand shift (due to world economic growth) and by supply shocks (Arab embargo and Iran-Irak war) which have moved the system from an old low-priced equilibrium to a new high-priced one (Gately, 1984).

However, if price changes are explained in terms of equilibrium move-

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ments there is no room for persistent or undamped oscillations. Moreover, this approach appears dynamically unsatisfactory because the equilibria occurring along the backward bending portion of the OPEC supply curve are not necessarily asymptotically stable. That being the case the price paths cannot be understood as trajectories going from the old stationary state to the new one, but they can be better thought of as fluctuations out of equilibrium.

An additional purpose of this article is to stress the role of the speeds of response to a disequilibrium. In fact, as it will be shown below, very different dynamic behaviours are generated by fast or slow price adjustments as well as by quick or reluctant quantitative reactions to a market excess.

2. *The Model*

In what follows we assume that world oil demand is a smooth function $d(p)$, with $d'(p) < 0$, $d''(p) < 0$, $d(\bar{p}) = 0$ for $\bar{p} > 0$, and $d(p)$ tending to infinity when p tends to zero. As to the supply, according to the available evidence (Griffin, 1985) different hypotheses are considered about OPEC and non-OPEC producers. The latter act as price takers and offer increasing quantities as price increases. Since it is natural to assume a technical maximum for production, their supply will be described by a smooth function $g(p) < \bar{x}$, with $g(0) = 0$, $g'(p) > 0$, $g''(p) < 0$ and with $\bar{x}$ being the full exploitation level of production.

The OPEC behaviour is more controversial (see Griffin-Teece, 1982; Gately, 1984), since its explanation swings between the competitive approach and the collusive one.

In spite of the fact that the present state of the theory seems to lack a well founded basis, we choose provisionally to adopt here the competitive approach. Furthermore, since many public statements of OPEC ministers and some statistical evidence seem to support the target revenue hypothesis, at least in its partial version (Griffin, 1985), both the standard upward sloping supply curve and the backward bending one are considered.

It is well known that Teece's original version of the "backward bending" case is rather questionable; and some recent works have tried to improve it, suggesting a producers' goal which takes account of average costs (assumed to be U-shaped). This approach gives rise to an isorevenue curve along which two quantities satisfy the target for any given price. However one can make the hypothesis that only the smaller quantity is actually supplied, so that a larger amount of the non reproducible resource is saved (Ruiz, 1986).

Assuming moreover that below the minimum price supporting the target the relevant supply is given by the marginal cost curve, and leaving aside problems of continuity to semplify the discussion, one can take an OPEC's supply function, $s(p)$, such that $s'(p) \leq 0$ for $p \geq \hat{p}$ (and by hypothesis $\hat{p} < \bar{p}$). Obviously unlike Teece's treatment, $s(p)$ is no longer a hyperbola; however the quantity supplied still tends to zero when price goes to infinity.

A further feature of the present model is an assumption of market sluggishness. This is usually explained by the existence of long term contracts which do not allow instantaneous price revisions. Thus the speed of response of the price to the global market excess, say $\theta_1 > 0$, crucially depends on the producers attitude to revise prices (or to offer in the spot market rather than sell by means of long-term contracts).

On the other hand, empirical evidence seems to suggest that non-OPEC countries do not adjust immediately their current production, x, to the desired level $g(p)$, so that their actual supply changes with a finite speed of reaction to price movements, say $\theta_2 > 0$.

Under the previous hypotheses the out-of-equilibrium dynamics of oil market can be described by the following two autonomous differential equations:

$$\dot{p} = \theta_1 [d(p) - s(p) - x] = \theta_1 [f(p) - x) \tag{1}$$

$$\dot{x} = \theta_2 [g(p) - x], \tag{2}$$

where the dots stand for derivatives with respect to time, and $f(p) = d(p) - s(p)$ is a C^1 function.

Figures 1 and 2 show the possible equilibria in the normal static demand-supply picture and in the phase portrait respectively. The latter shows the loci z and v for which it is $\dot{p} = 0$ and $\dot{x} = 0$ respectively.

Three alternative OPEC supply curves are drawn: the first one depicts the usual upward-sloped case; the other two show the backward-bending one. In particular S_3 generates multiple equilibria.

An equilibrium, say (p^*, x^*), occurs when it is $x = g(p) = f(p)$, i.e. when the actual non-OPEC production is equal to both the independent producers optimal supply and the amount $f(p)$ that OPEC plans to leave them in order to satisfy its target. For the sake of simplicity we assume the local uniqueness of (p^*, x^*), which amounts to excluding a case of tangency in the phase space.

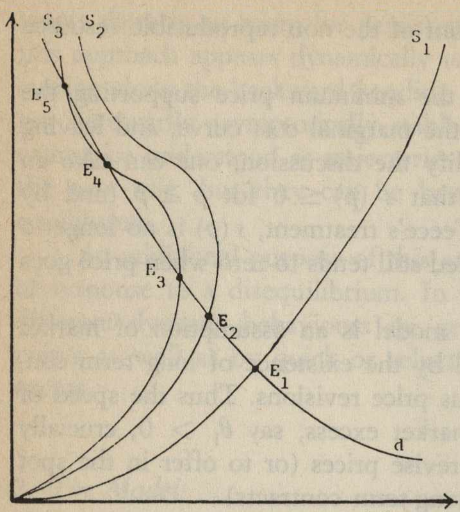

FIGURE 1.

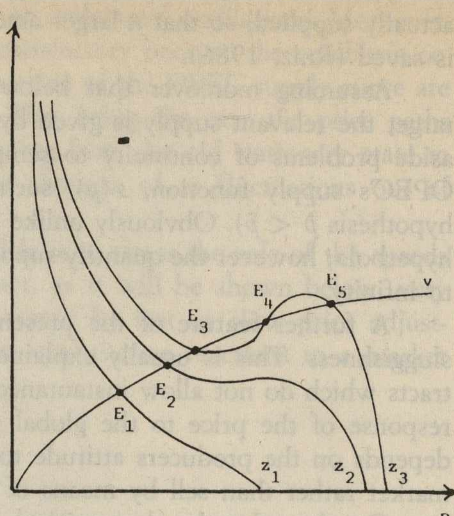

FIGURE 2.

The (local) disequilibrium behaviour of (1)-(2) can be analysed by looking for the characteristic roots of the auxiliary equation

(3) $$\lambda^2 - (\theta_1 f' - \theta_2)\,\lambda + \theta_1\theta_2\,(g' - f') = 0,$$

where the derivatives are evaluated at the equilibrium (this is the same as finding the eigenvalues of the Jacobian matrix associated with the system linearized around p^* and x^*).

The system is locally asymptotically stable if and only if both roots of equation (3) have a negative real part, that is if

(4) $$g' - f' > 0$$

(5) $$\theta_1 f' - \theta_2 < 0.$$

Given that $g' > 0$, the above conditions hold *necessarily* if we have $s' > 0$ (and obviously $f' < 0$), that is when the equilibrium occurs along the normally bending branch of the supply curve (like E_1 or E_2). Then the dynamic behaviour of the market is quite usual. If, for example, the current supply exceeds the world demand at a given price, the latter falls. Accordingly the producers reduce their supply, while the demand increases and the process goes on until the equilibrium is attained. The actual levels of p and x approach their stationary levels, exponentially or spiralling with damped oscillations according to whether:

(6) $$(\theta_1 f' - \theta_2)^2 > 4\theta_1\theta_2 (g' - f').$$

The same conclusion would be reached if $s' < 0$ and $f' = d' - s' > 0$, with (4) and (5) holding. These conditions amount to say that around the equilibrium, although the OPEC supply curve is backward bending, the global one is *either* normally bending *or* downward sloped but steeper than the global demand curve.

For instance, let us suppose that the previous conditions are satisfied at E_3 and E_5. Since at the intermediate equilibrium condition (4) is reversed, the auxiliary equation has two real solutions with opposite sign so that E_4 is a saddle point. Therefore the price and the production are pushed away unless, by a mere fluke, their initial values lie exactly on E_4 or on its unique stable manifold.

The story is well known in the literature (Gately, 1984). When around (below) the intermediate equilibrium there is a supply excess, p and x fall down until the low-priced one is eventually reached. On the other hand when near (above) E_4 there is a demand excess, p and x increase (while OPEC production shrinks) until the high-priced equilibrium is attained. According to some recent interpretations of the target revenue theory (Gately, 1984), it is precisely the latter case that the supply shocks of the seventies would have brought about. However it has to be noticed that in Teece's early version the upper stable equilibrium is not mentioned at all. This seems to imply that, during the recent high-price period, the oil market should have been on a sort of pin-head equilibrium for ten years.

3. *Persistent Oscillations*

Let us consider an equilibrium at which $g' > f' > 0$ (like at E_3) but the inequality (5) does not necessarily hold. In this case the dynamic behaviour of p and x also depends on the speeds of response.

In fact if it were the case that $\theta_1 f' > \theta_2$ the roots of (3) might be complex with non negative real part, and the market oscillations could be undamped.

By exploring the above condition one gets a straightforward economic interpretation. In fact, given that now $s' < 0$, the inequality $\theta_1 f' > \theta_2$ implies that $d'(p)$ is low enough (i.e. the demand rigid enough) for f' to be positive, and also that θ_1 is high enough and θ_2 small enough for $\theta_1 f' - \theta_2$ to be non-negative.

Therefore, in general, a high rigidity of world demand, together with a fast price responsiveness and a slow non-OPEC reactivity to a disequilibrium, work in favour of persistently fluctuating regimes. This conclusion is sensible when one thinks of the recent and severe downward price fluctuation, which seems to have been caused by a sudden change in the attitude to revise prices downward, especially on the part of Saudi Arabia, while non-OPEC producers tried to keep their production levels unchanged. As all this belongs to the recent past, it is not difficult to explain how it happened leaving some political and tactical aspects aside.

If oil price falls very quickly in response to a global supply excess (θ_1 high) and non-OPEC producers are loath to promptly adjust their share downward (θ_2 small), then the price overshoots its equilibrium level and keeps going down. Consequently the market excess is cleared when $p < p^*$; it is only then that p starts rising again, but it goes beyond its stationary level once more. Undamped or explosive up and down movements may continue to take place as long as relative speeds remain such that (5) does not hold.

Indeed, actual oil market fluctuations are irregular and bounded. This could be modelled only by means of a very complex, more than two-dimensional, system (which *inter-alia* would allow to draw a more realistic distinction between the "cartel core" and the expansionist fringe of OPEC).

But this is something that falls beyond the scope of the present paper. However some insights on the boundedness of oscillations can be obtained as well from the present model. More precisely it is possible to prove the existence of closed orbits, that is of bounded cycles, by means of a bifurcation theorem. In fact the dynamic system (1)-(2) is C^1 at least, as a consequence of the assumptions made about $d(p)$, $s(p)$ and $g(p)$; moreover it possesses two purely imaginary roots at $\theta_1 = \bar{\theta}_1 = \theta_2/f'$ (or $\theta_2 = \bar{\theta}_2 = \theta_1 f'$). Since, in addition, the characteristic roots vary continuously with θ_1 (or θ_2), θ_2 (θ_1) being fixed, their real part change with non zero speed when the parameters cross the "bifurcation level" $\theta_1 = \bar{\theta}_1$ (or $\theta_2 = \bar{\theta}_2$). So the Hopf bifurcation theorem applies (Alexander-Yorke, 1978).

The attractiveness or repulsiveness of the closed orbits cannot be ascertained without knowing the higher order derivatives of $f(p)$ and $g(p)$.

Nevertheless one could prove the existence of a limit (stable) cycle by means of the Bendixon-Poincaré theorem.

For the sake of simplicity let us consider a unique equilibrium (depicted in Fig. 3), say E, such that $g' > f' > 0$, but the inequality (5) is reversed. This means that around it the roots of (3) both have positive real parts i.e. the equilibrium is unstable.

Now let us consider the compact set bounded by the line $ABCD$. The segments AB and BC belong to the vertical and to the horizontal axis respectively. Obviously on the former it is $p = 0$, so that from (1) one gets $\dot{p} > 0$; on the latter, being $x = 0$, it will be $\dot{x} > 0$, according to (2). AD satisfies the equation $x = \bar{x}$, which implies $\dot{x} < 0$ since, for any finite p, it is $g(p) < \bar{x}$. Finally DC has $p = \tilde{p}$ for equation, where $\tilde{p}$ is a level of oil price for which $f(\tilde{p}) < 0$: therefore, looking at (1), we see immediately that $\dot{p} < 0$ must hold there.

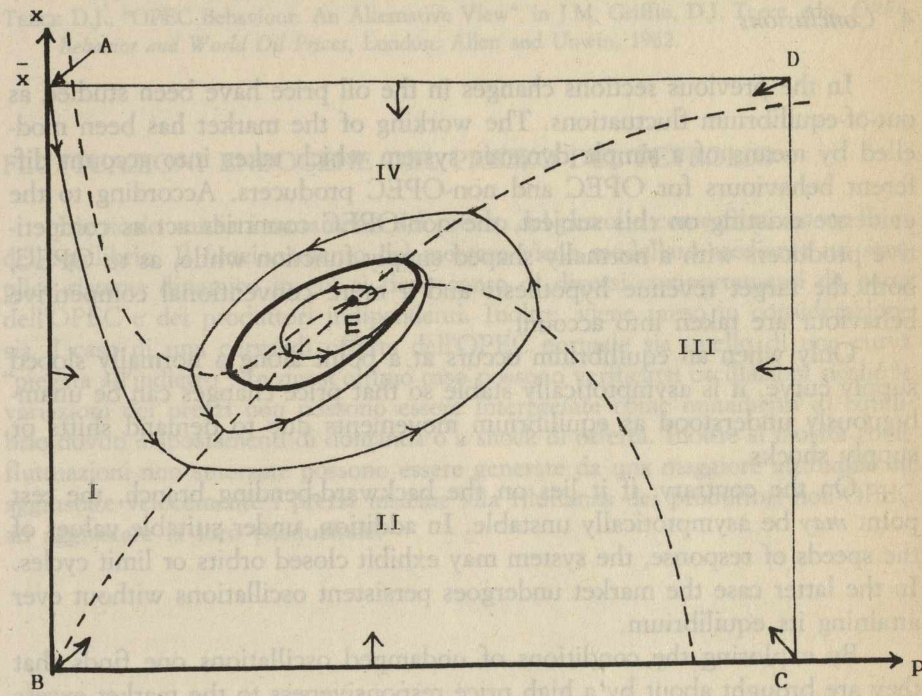

FIGURE 3.

Thus the area bounded by $ABCD$ is a non empty compact set in (x, p) space such that the dynamic system (1)-(2) points inwards on the boundary. Under the conditions specified above, the Bendixon-Poincaré theorem implies the existence of at least one limit cycle (Andronov-Vitt-Khaikin, 1966).

The graph in Fig. 3 shows the behaviour of the system out of E. For instance let us take a point (an initial condition), say A $(\bar{x}, 0)$, belonging to the boundary. The trajectory or phase path starting from it moves through the region I (where $\dot{p} > 0$ and $\dot{x} < 0$) until it cuts the locus v below E. Successively it spirals counterclockwise without ever intersecting itself.

On the other hand any path starting from E moves away possibly spiralling. By the theorem of uniqueness of the integral curves passing through non singular points, the two trajectories cannot intersect, but both tend to a closed orbit (not necessarily unique) which is a limit cycle.

4. *Conclusions*

In the previous sections changes in the oil price have been studied as out-of-equilibrium fluctuations. The working of the market has been modelled by means of a simple dynamic system which takes into account different behaviours for OPEC and non-OPEC producers. According to the evidence existing on this subject, the non-OPEC countries act as competitive producers with a normally shaped supply function while, as to OPEC, both the target revenue hypothesis and a more conventional competitive behaviour are taken into account.

Only when an equilibrium occurs at a point along a normally sloped supply curve, it is asymptotically stable so that price changes can be unambiguously understood as equilibrium movements due to demand shifts or supply shocks.

On the contrary, if it lies on the backward-bending branch, the rest point *may* be asymptotically unstable. In addition, under suitable values of the speeds of response, the system may exhibit closed orbits or limit cycles. In the latter case the market undergoes persistent oscillations without ever attaining its equilibrium.

By exploring the conditions of undamped oscillations one finds that they are brought about by a high price responsiveness to the market excess on one side and by the non-OPEC reluctance to adjust the production on the other. This seems to be a sensible interpretation of recent events and underlines the importance of using a dynamic approach to describe the oil market behaviour.

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FLUTTUAZIONI ENDOGENE DEL PREZZO DEL PETROLIO

L'articolo studia i mutamenti del prezzo del petrolio come fluttuazioni fuori dell'equilibrio. Il funzionamento del mercato viene modellato mediante un semplice sistema dinamico in cui si tiene conto di diversi comportamenti da parte dell'OPEC e dei produttori indipendenti. Inoltre, viene preso in considerazione sia il caso di una curva di offerta dell'OPEC normale sia quello di una curva "piegata all'indietro". In quest'ultimo caso possono verificarsi oscillazioni sicché le variazioni dei prezzi non possono essere interpretate come mutamenti di equilibrio dovuti a spostamenti di domanda o a shock di offerta. Inoltre si mostra come fluttuazioni non smorzate possono essere generate da una maggiore attitudine ad aggiustare velocemente i prezzi insieme alla riluttanza dei produttori non-OPEC ad aggiustare la loro produzione.

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ON THE DUAL RELATIONSHIP BETWEEN FLEXIBLE ACCELERATOR AND q THEORIES OF INVESTMENT

by
MARZIO GALEOTTI *

Flexible accelerator and Tobin's q models are nowadays the two leading theories of investment, competing with each other both in the theoretical literature and in applied work.

The flexible accelerator model was originally derived by Jorgenson (1963) from a choice-theoretic problem of the firm's behavior. It is well-known and now widely accepted that there were certain inconsistencies in Jorgenson's approach: these have been subsequently remedied through the introduction of costs of adjustment into the analysis [1]. The result is a model fully consistent with the underlying optimization problem for the firm, which has been termed "rational" flexible accelerator (Treadway, 1971).

The q theory stems from Tobin's (1969) research on the structure of the financial sector of the economy. Empirical investment equations have been implemented by constructing average q, the ratio between the market valuation and the replacement cost of the firm's capital assets, from financial markets and balance sheet data (e.g. Tobin and Brainard, 1977). Not until recently has the model been provided with rigorous theoretical foundations in terms of the firm's rational behavior. Abel (1979) has shown that the q model can be obtained from the *very same intertemporal optimization problem incorporating adjustment costs, employed to derive the rational flexible accelerator.* q is identified with the ratio of the shadow to the market price of newly installed capital goods. Furthermore, if the firm's technology-cum-adjustment cost is linear homogeneous, marginal and average q coincide (Hayashi, 1982). Thus, the use of the latter in empirical work is rigorously

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[1] See GALEOTTI (1984) for a survey.

justified, thereby remedying the fact that it is the former which is relevant for the theory but only the latter is observed. The upshot of these developments is that also the q model is now a structural model with firm theoretical foundations.

Both flexible accelerator and q models can be seen as stemming from a common underlying optimization framework where adjustment costs play a crucial role. In their most common specification, the models can be written as follows:

$$\dot{K} = \theta (K^* - K) \tag{1}$$

$$\dot{K} = h (q - 1) \tag{2}$$

where θ is the (optimal) speed of adjustment of the capital stock to its desired level K^*, and h is the (inverse of the) marginal cost of adjustment.

By looking at (1)-(2), it appears that *both models offer the same qualitative explanation of the investment mechanism*: this is put to work whenever a discrepancy between the target and the actual value of the key determinant is perceived. Comparing the two formulations, it also emerges that the disequilibrium [2] that characterizes both models is looked at from a different perspective: the q approach underscores a *price adjustment process* [3], while the flexible accelerator approach emphasizes a *quantity adjustment mechanism*. Furthermore, it also appears that the coefficients θ and h perform the same function and deliver the same type of information: that of determining the pace at which investment optimally evolves as actual capital stock approaches its long-run desired level on the one hand, and as q reaches its equilibrium value (here taken for simplicity to be one) on the other.

The analogy between the two theories has been noticed by Bosworth (1975):

> Although it appears much different, the securities-valuation model is conceptually identical to the version of the neoclassical model used in most empirical studies (p. 285).

More explicitly, Tobin and Brainard (1977) observe:

> The hypothesis that investment is related to the difference between R (margi-

[2] This terminology is loose: this is a dynamic equilibrium situation, in the sense that investment evolves over time along an optimal path.

[3] This emerges more clearly from a formulation of the optimization problem in which adjustment costs are internal to the firm's technology. In this case the q model has the following format: $dK/dt = h(q - g)$, where q is the shadow price and g the market price of new capital goods. See Galeotti (1986).

nal efficiency of capital) and r (interest rate used to discount future earning streams), or the value of q, bears some resemblance to the "flexible accelerator" idea that investment is a function of the difference between a desired and actual capital stock (p. 245).

Given that a duality appears to exist between the two models, it ought to be possible to establish a precise analytical link between them, taking into account their common theoretical framework [4]. The formal derivation of a general relationship between q and flexible accelerator models seems however difficult, mainly for the impossibility of solving explicitly for the long-run demand for capital K^*, unless specific functional forms for the technology are postulated. In addition, the analysis underlying the rational flexible accelerator is in general only locally optimal, whereas that is not the case for the q theory. Nevertheless, the existence of a precise link can be shown under the hypotheses of a somewhat specialized class of models. To this purpose, I will adopt here Gould's (1968) model featuring a quadratic external adjustment cost function depending on gross investment, constant returns to scale, and static price expectations. As demonstrated by Treadway (1974), such model belongs to the class that generates globally optimal flexible accelerator mechanisms with a constant optimal speed of adjustment.

Let the problem be:

$$\max V = \int_0^\infty \exp(-rt)\{\pi(K, w) - g[I + c(I)]\}\, dt$$

where the variable profit function, owing to constant returns, is such that:

$$\pi(K, w) = K \cdot \rho(w); \qquad \partial\pi/\partial K = \rho(w)$$

Furthermore, the unit adjustment cost function takes the form:

$$c(I) = (1/2\alpha) \cdot I^2$$

[4] A rationalization of this fact is attempted by CICCOLO and FROMM (1979). In a model where there are no adjustment costs, they establish the result: $qK = K^*$. They then *postulate, but not derive*, the flexible accelerator model: $dK/dt = \lambda(K^* - K)$, so that by substitution: $(dK/dt) \cdot (1/K) = \lambda(q - 1)$. Unfortunately, this result is incorrect. Firstly, in absence of adjustment costs the rational flexible accelerator does not obtain, as the maximization problem delivers neither an optimal rate of investment nor an endogenous adjustment coefficient. Hence the criticism to Jorgenson's approach (e.g. BRECHLING, 1975 and NERLOVE, 1972) applies to their analysis. Secondly, to obtain their relationship between q and K^* they assume profit maximization and constant returns to scale. But it is well-known that under such conditions a steady-state or desired capital stock is not defined. Hence the expression $qK = K^*$ is not valid.

where α is a positive constant [5]. The optimality conditions are:

$$\mu = g\,[1 + c'(I)] = g\,(1 + I/\alpha) \tag{3}$$

$$\dot{\mu} = (r + \delta)\,\mu - \rho\,(w) \tag{4}$$

$$\lim_{t \to \infty} \mu \exp(-\,rt) = 0 \tag{5}$$

Using (5) when integrating (4) yields:

$$\mu = \rho\,(w)/(r + \delta) \tag{6}$$

Solving (3) for I:

$$I = \alpha\,(\mu/g - 1) \tag{7}$$

Defining marginal q as the ratio of the shadow price μ to the market price of new capital g, (7) represents our q model of investment:

$$I = \alpha\,(q - 1) = \alpha\,[(\rho\,(\omega)u) - 1] \tag{8}$$

where $u = g\,(r + \delta)$ is the user cost of capital.

Turning to the flexible accelerator, substitute (3) into (4), differentiate over time (noting that $\dot{I} = \ddot{K} + \delta\dot{K}$) to obtain:

$$\ddot{K} = r\dot{K} + (r + \delta)\,\delta K - (\alpha/g)\,[\rho\,(w) - u] \tag{9}$$

This differential equation can be solved exactly. The characteristic roots are $\theta_1 = r + \delta > 0$ and $\theta_2 = -\,\delta < 0$. Ruling out the former as a consequence of (5), after some by now familiar manipulations we obtain the rational flexible accelerator model:

$$\dot{K} = \theta^*\,(K^* - K) = \delta\,(K^* - K) \tag{10}$$

where the long-run capital stock K^* is obtained from (9) by setting $\ddot{K} = \dot{K} = 0$:

$$K^* = (\alpha/\delta)\,[(\rho\,(w/u) - 1] \tag{11}$$

The similarity between (7)-(8) and (10)-(11) is already evident. In particular:

$$K^* = (\alpha/\delta)\,(q - 1) \tag{12}$$

[5] It should be stressed that the quadratic form is not necessary for our results, but it is adopted for expositional clarity.

Hence:

$$\dot{K} = \theta^* [(\alpha/\delta)(q - 1) - K] \tag{13}$$

It is interesting to note that α in (12) stands for h in (2): it follows that q and K^* are related to each other by the ratio between the (inverse of the) marginal adjustment cost and the speed of adjustment coefficient. That is, by precisely the terms that set the pace of optimal investment in the two theories.

For a fairly general class of models, an exact analytical relationship between q and K^* has been shown to exist in the context of a consistent choice-theoretic framework for the firm [6]. The presumption that there is a conceptual link between optimally derived flexible accelerator and q models of investment is confirmed and it is likely to hold, at least approximately, in the case of more general theoretical settings. Whether it is actually possible to establish more general conditions under which the above duality relationship holds, is an open question and a topic left for future research.

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SULLA RELAZIONE DUALE TRA ACCELERATORE FLESSIBILE E TEORIA q DI INVESTIMENTO

Questa nota mostra come i modelli di acceleratore flessibile e q di investimento siano l'uno il duale dell'altro: raccontano entrambi la stessa storia, sebbene con una diversa prospettiva, sull'attività di investimento delle imprese. Si dimostra, in particolare, che per una classe di modelli abbastanza generale esiste una relazione analitica esatta tra le due teorie.

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THE NINETEENTH CENTURY DEVELOPMENT OF THE RUSSIAN PETROLEUM INDUSTRY

by
JOSEPH A. MARTELLARO *

In his *The Enigma of Soviet Petroleum*, Marshall I. Goldman has succinctly written:

> There is something about petroleum that is controversial and intriguing. There is something about Russia that is mystifying and absorbing. When the two merge in a study of Russian petroleum, the result is likely to be tantalizing and engrossing [1].

Indeed, such is the case, for there appears to be no dearth of literature – economic and non-economic – dealing with the Soviet petroleum industry, past and present. However, the very abundance of literature, contrary to what one might surmise, does not necessarily constitute a mass redundancy of substance, for although serious students and scholars researching the Russian petroleum industry have often reached similar or identical conclusions, the published literature in the field is also characterized by variety and diversity. Clearly, while traversing their roadways of research, researchers have found their points of departure [2].

Just prior to the turn of the century, Russia surpassed the United States as the world's leading producer of crude oil – accounting for one half of the world's output. After relinquishing its position of leadership to the USA in 1902, not until 1974 – when Soviet output equalled 458.9 million

* Northern Illinois University, Department of Economics, DeKalb, Ill. (U.S.A.).

[1] GOLDMAN (1980, p. 13).

[2] As an example, fine scholars such as Fred Carstensen, Olga Crisp, Alexander Gerschenkron, John P. McKay, Henry Rosovsky, and Theodore Von Laue hold their respective views as to the extent to which private industry – as opposed to state interventionism – contributed to the development of the Russian petroleum industry.

metric tons to 432.8 for the USA – did the USSR regain world primacy as a producer of petroleum. In a large measure, the foundation of the present-day Soviet oil industry was laid during the last quarter of the nineteenth century. The purpose of this paper is to (1) focus on the formative years of the Russian oil industry which, by and large, were concentrated in the Baku, and (2) devote some attention to a number of the individuals who were instrumental in the development of Russia's nineteenth century oil industry.

The Early Years

Although they were not aware of its potential economic value, the seventeenth and eighteenth century residents of the area – which in time came to be known as the Baku – were cognizant of the presence of natural gas and petroleum and its use. Less than ten miles outside of the city of Baku, there still stand the partial ruins of a temple; a site where the Parsees worshiped fire. The perpetually burning flames at the temple were fueled by the natural gas escaping from the pockets directly below the holy site. It is believed that the fire-worshipers (followers of Zoroaster) had originally emigrated from India.

Although the Baku had been conquered by Peter I in 1723, it was taken by the Persians some twelve years later. During the reign of Nicholas I, by absorbing the Baku Khanate in 1813, the Russian State won the petroleum rights to the Baku, a land tract on the Aspheron Peninsula extending into the Caspian Sea. In recent years, scholars have referred to this area as the "First Baku" [3]. Prior to the Russian seizure of the Baku, the residents extracted crude oil simply by the use of rags and buckets; a primitive technique, but given the adundance of petroleum, no better method was necessary.

Understandably, the first half of the century witnessed very limited exploitation of the rich oil fields of Russia. But with the ending of the Crimean War (1854-56), government intervention to foster industrial development – including the petroleum industry – began in earnest. The Crimean War, not unlike other wars to follow, awakened the nation to its industrial lethargy and economic backwardness in contrast to the nations of western Europe.

[3] Not until after World War II did the Soviets, in a large measure, exploit the "Second Baku", an area north of the Caspian Sea which stretches from the Urals to the Volga River. Presently being developed are the "Third Baku" area (in Soviet Central Asia), western Siberia, and some regions in the far east.

The Baku Fields and Tsarist Policies, 1821-1900

With the exception of a few scattered locations, oil production from 1870-1913 was restricted in the Baku area. Until the 1870s, growth in Russia's petroleum industry was negligible, but thereafter, the industry was vitalized by an inflow of foreign capital. Although foreign investors directed their resources primarily into refining and secondarily into production; nevertheless, as a consequence of their capital investments, the oil industry became the nation's fastest growing industrial sector. In 1883, slightly more than 1 million metric tons of oil were produced in comparison to only 4,000 thousand metric tons in 1860. From 1813 to 1873, in exploiting its petroleum resources, the state alternated its policy between a monopoly-contract-system and state intervention.

However, during the critical growth years of the petroleum industry, 1860-83, the government for the most part abandoned its policy of direct state intervention in industry. In contrast to prior years, one observer described the economic milieu of the Baku in 1883 as follows:

> There is not an industry in Russia to-day where the *laisser faire* doctrine is carried to such lengths as in the Baku petroleum trade, and in this respect it will stand comparison not only with that of Galicia, but with the freest portion of the United States' oil fields [4].

The dramatic shift to a free enterprise oil industry was the result of several government inquiries led by V.A. Kokorev, a Muscovite entrepreneur, and Dimitrii Mandeliev, a respected chemist. The group conducting the investigation had concluded that unfettered entrepreneurship was the answer to what had become an oil industry [5]. The reaction by entrepreneurs was swift and positive.

In no small way, the industrial policies of the state were shaped by the Tsar's ministers, and among them, the Minister of Finance was undoubtedly the most influential in the shaping of economic policy. Theodore H. Von Laue has well described the powerful role of the Finance Minister.

> Of all the ministers of the tsars the minister of finance bore the brunt of official invidious comparison between Russia and the other great powers of Europe. More than any other government servant he had to assume responsibility for Russian poverty and the search for an escape from it. His function in the gov-

[4] MARVIN (1884, p. 209).

[5] *Ibidem* (1984, pp. 606-07).

ernment thus became by necessity ever more crucial. He was in charge of the purse and presumably he could call the tune, at least how funds were to be secured, and perhaps also how they were to be divided. Under the right conditions he might well become the real master of the government or even of Russian society [6].

Four powerful Ministers of Finance were instrumental in the formation of Russian industry during the nation's first three decades of commercial development. Mikhail Khristoforovich Reurten (1862-76) served during the embryonic stage of the oil industry; he was succeeded by Nicolai Khristanovich Bunge in 1881. In 1887, Alexander III appointed Ivan Aleksievich Vyshnegradskii to succeed Bunge who had resigned in the previous year. In turn, Vyshnegradskii was replaced by the very competent Sergei I. Witte in 1892. Each of these ministers, in his own way and to a different degree, strived to improve on state finances, attract foreign investment, and stabilize the ruble. Furthermore, each played a role in the growth and development of the oil industry during its formative years of the century, 1870-1900.

During his tenure, Vyshnegradskii adopted a fiscal policy of budgetary restraint, a strict style of financial management. His austere measures earned for him the distinction of becoming the first Minister of Finance since the Crimean War (1854-56) to consistently formulate budgets which balanced or yielded surpluses. Moreover, Vyshnegradskii was dedicated to the task of harnessing the nation's resources in order to bring about rapid industrialization, including the oil industry. His deliberate planning, sound fiscal and monetary policies, plus his highly protectionist Tariff Act of 1891 gave various budding industries both the impetus and the valuable time required to develop. Consequently, his skill in directing the nation's industrial development during the period of 1885-89 was instrumental in the achievement of an industrial growth rate of 6.1 percent.

The valuable work of Vyshnegradskii did not flag upon his leaving office, but as indicated above, he was succeded by the ambitious and energetic Sergei I. Witte who – by utilizing Vyshnegradskii's accomplishments as a springboard – achieved even greater heights in the promotion of Russian industry. During the final decade of the century, industrial growth had climbed to a spectacular 8.03 percent, unmatched by any other industrialized nation. Thus, during Witte's tenure, the hand of government was most visible in matters related to industrial development, including the oil industry.

Some scholars of nineteenth century Russia consider state intervention

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[6] Von Laue (1963, p. 4).

as the most significant factor in bringing about the industrial surge which took place in Russia during the 1890s, e.g., in the production of coal, pig iron, petroleum, and steel. The government's expansion of the railroad system and other infrastructure lent support to the nation's movement toward industrial growth; as an example, construction of the Baku-Batum railway which became operational in 1883. Linking the chief producing areas on the Caspian to the Black Sea, the new railroad line speared the way for a rise in oil exports [7].

With the exception of the 1860-83 period discussed above, the Russian Crown was reluctant to permit the oil industry to naturally develop within a free market situation, a posture contrary to that of several western nations. As is commonly recognized, unlike the Russian case, the American business environment was probably the most liberal among nations of the West. In the American nineteenth and early twentieth century setting, the adventurous, individualistic, and risk-taking entrepreneur fitting the Joseph A. Schumpeterian mold was ideally suited to assume a major role in bringing to fruition the potential of the infant oil industry of America. In the light of the American experience, one might persuasively argue that the mushrooming of Russia's new industrial center in the Caucasus during the latter part of the nineteenth century may well have occurred in spite of government intervention, not because of it. And although the Russian oil industry enjoyed a satisfactory rate of growth during the four decades bridging the two centuries (1870-1913), the rather restrictive policies adopted by the Russian throne during those important years of gestation may have well braked the industry's bid to attain even greater heights of prosperity.

For almost a century, three different means of allocating oil lands were employed by the Tsars and their various finance ministers:

1. The Lease-system, 1821-72 [8].
2. The Auction-system, 1872-96.
3. The Auction/Royalty System, 1896-1917.

By the Lease-system, government declared the exploitation of oil a state monopoly thereby reserving for itself the power to lease the fields; all leases were limited to a term of four years. The brevity of the lease proved to be a serious disadvantage to the lessee, for the limited technology of the day required that a good part of the four years be devoted to drilling,

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[7] In 1874, a special commission had recommended the extension of the Poti to Tiflis Railroad, but not until after the inadequacy of the railroad was shown by the Russo-Turkish War plus the oil industry's expressed need for better transportation was the Poti-Tiflis Railroad extended to the Baku.

[8] For a paper dealing exclusively with the Lease-system, see MARTELLARO (1985).

construction, and set-up, leaving a limited amount of production time at the rump-end of the four-year lease. As a consequence, the state treasury derived only a modest income from the leases, revenues totaled but 3.74 million rubles ($ 3 million 1914 gold dollars) during the fifty years the system was in force.

Because of the termination of the lease system by Alexander III in December 1872, the oil fields were publicly auctioned off and transferred to private owners as semi-permanent leaseholds. Small plots, measuring up to ten dessiantines (27 acres), were auctioned to the highest bidders; however, the parcels were auctioned in single lots to avoid the formation of large-scale operations by the purchasers, either through joint-ventures or severally. On the whole, the Auction-system was probably an improvement over the Lease-system, but some entrepreneurs considered the new terms just as unattractive as those offered under the old system.

With the appointment of Count Witte as Minister of Finance in 1892, the Auction/Royalty System was implemented, a system which was in force until Russia's withdrawal from World War I. By the A/R System, the government sought to increase its revenues not only by auctioning the fields but also by requiring the payment of a royalty. The amount of royalty paid was based upon a percentage of the value of the oil produced by a firm. In some instances, the state royalty was exceptionally high – as much as 40 percent. Not surprisingly, the unfavorable terms imposed by the government became a factor which dulled the incentive of producers [9]. In the light of the flagging growth, Russia was unable to retain its position of world primacy attained in 1898.

Late Nineteenth Century Growth, 1870-1900

Despite the rather strict controls imposed upon the oil industry by the tsars and their finance ministers, the industry did develop and grow throughout the last three decades of the nineteenth century. Notwithstanding some early signs of imminent reverses, the oil industry performed quite

[9] Other at least equally influencing factors accounting for a setback in the oil industry shortly after the turn of the century were the (1) industrial labor strife which began prior to 1900 and continued into the early 1900s, (2) unfavorable water-to-oil ratio which for some time had been progressively developing at the wells, (3) depletion of the high horizon wells requiring the deepening of many wells from 300 to 600 feet and the drilling of new ones, (4) failure of the oil producing firms to import new, improved technology, and (5) cut-throat competition which drove many oil firms into bankruptcy.

well during the last decade of the century, for the growth index reached 275 in 1900 compared to the base year of 1890 [10]. Table 1 shows both the absolute and relative increases in Russian oil production from 1885 to 1899. The data provided in Table 1 reflect quite impressive growth during the fifteen-year period, sufficient to provide the momentum needed to propel Russia into its position world leadership at the turn of the century.

TABLE 1

ANNUAL AVERAGE OIL OUTPUT, 1885-1899
(Expressed in Millions of Metric Tons)

Period	Output[a]	Percentage Increase
1885-89	2.77	——
1890-94	5.09	83.75
1895-99	8.25	62.10

(a) Output in the original text was expressed in poods, a Russian unit of weight. That unit of measure has been converted to tons.

Source: FALKUS, (1972, p. 60).

The vigorous development and growth of the oil industry is ascribable to several factors. First, despite government intervention and strict policies, foreign capital flowed reasonably well into the country in support of Russian industry. By 1890, foreigners had invested 215 million rubles in Russian industry, and during the subsequent five years, aggregate investment rose to 280 million rubles. By 1900, foreign investment totaled 910 million rubles, an increase of 322 percent over the last five years of the century [11]. In descending order, Great Britain, France, and Germany were the principal sources of foreign investment funds.

Second, as mentioned earlier, the Russian defeat at Sevastopol during the Crimean War compelled Russia to recognize its economic backwardness compared to nations in the west. This neo-awareness to its economic defi-

[10] This phenomenal growth rate was not peculiar to the oil industry. Over the same period of time, indices for coal, pig iron, and steel measured 269, 314, and 586 respectively. FALKUS (1972, p. 76).

[11] *Ibidem* (1972, p. 70).

ciencies brought a new wave of westernization; western capital, technology, and brainpower. Third, by interrupting oil supplies from the United States, the Russo-Turkish War (1877-78) forced the growth and development of the Russian oil industry, for domestic producers seized the opportunity to exploit the major share of their home markets.

Fourth, although it marked a return to state intervention in industry, the ascension of Count Witte to the Ministry of Finance in 1893 was a factor which contributed to the expansion of the oil industry. His financial-industrial expertise recognized the benefit to be derived from an expanding industrial sector. One of the first steps toward promoting industrial growth was a program providing massive state credits of over a billion rubles to supplement foreign capital flowing into the nation. And although oriented toward fostering the growth of several industries, considerable benefits were also bestowed upon on the oil industry.

Fifth, the serious attention given to the improvement of the infrastructure provided external economies to the oil industry. Sixth, despite several of the aforementioned disadvantageous features of the Auction-system of 1872-96, by the end of the century, the oil industry had become oligopolistic; by 1900, three major oil companies accounted for 90 percent of the oil output. By virtue of their size, the three companies were able to amass considerable resources to further expand their holdings in the oil industry.

Seventh, nineteenth century Russia benefited by the presence of a number of foreign entrepreneurs willing to invest their time, money, and innate and acquired skills in an effort to win the potential profits offered by Russian markets. Also, Russian officials by attracting ambitious and highly motivated foreign entrepreneurs to the Russian industrial scene were able, through their example, to attract additional talented entrepreneurs. Among those recruited early were the Englishman John J. Hughes, who concentrated his efforts in heavy industry; the German Ludwig Knoop, a leader in the cotton textile industry by establishing some 120 cotton textile mills; and the Swedish Nobels, who were the vanguard in the development of the Russian petroleum industry.

The Nobel Family

In surveying the development and promotion of industry in nineteenth and twentieth century Russia, the Nobel name is frequently encountered. A family of well-educated, dynamic leaders, their inventive and innovative minds made possible the creation of myriad new industries. The Nobels were active in armaments, industrial tools, shipbuilding, steam engines, and transportation. But above all, the Nobel name remains indelibly impressed within the international annals of the Russian petroleum industry, to the extent that Robert W. Tolf has appropriately dubbed the Nobel brothers as the "Russian Rockefellers" [12].

Encouraged to do so by Lars Gabriel von Haartman, a Finnish official, Immanuel Nobel, an armaments expert, emigrated from Sweden in 1837 to seek his fortune in Russia. Heavily in debt due to several previous business failures, he nevertheless assumed new risks in Russia. Lucrative government contracts for armaments and steam engines brought him wealth during the reign of Nicholas I. The introduction of his famous underwater mine contributed to his great fortune which he lost under Alexander I. Bankrupt he returned to Sweden in 1859, leaving it to his older brother Ludwig to carry on in business.

Nevertheless, Immanuel's wealth made it possible for his sons (including Alfred B. of Nobel Prize fame) to not only derive the benefits often associated with private tutoring but to also profit from industrial "internships" arranged for them in – and outside of the family factories. In the light of these advantages, the family expertise took on three salient and advantageous dimensions: business administration and organization, economics, and technology.

In 1873, while in the Baku area to purchase lumber for Ludwig's factory, i.e., walnut wood suitable for the manufacture of rifle stocks, Robert's interests were diverted to several parcels of oil-rich land owned by the De Boer brothers. After some deliberation, Robert opted to use an entire sum of 25,000 rubles to purchase the oil land.

Thus, a fledgling oil firm was established in 1874, which in time distinguished Ludwig and Robert as pioneers in the Russian oil industry [13]. Until the creation of the Nobel oil firm, Robert had struggled to gain recognition in the financial world, but he was convinced that he could produce a kerosene whose quality would supersede others available at the time. A fairly competent chemist who had worked with kerosene before, Robert Nobel improved the De Boer equipment. By 1875, he was producing a quality of kerosene which was not only superior to that marketed by some 150 other refineries in the area but a kerosene which was also competi-

[12] Tolf's, *The Russian Rockefellers* (1976), is generally respected as a work addressing itself to the role of the Nobels in the development of the Russian oil industry. However, Tolf's work has not escaped criticism, as an example, see Fred V. Carstensen's review which appears in *Russian History,* Volume 4, Part 2 (1977, pp. 202-204).

[13] Tolf (1976, pp. 45-46).

tive to the fine burning fuel produced in the USA. (Despite the high Russian import tariff, the USA was supplying as much as 90 percent of the kerosene consumed in Russia between 1869 and 1872).

Given the rapidly rising demand for kerosene as a form of energy for the purpose of illumination, the Nobel's high-grade kerosene not only found a ready market domestically but in time it also superseded the American sales volume of kerosene in Russia. (Among the peasants kerosene came to be viewed as a necessity, thus adding impetus to an already expanding market). Furthermore, recognizing the market potential for *ostaski* (the dark residual which remains after distilling crude to kerosene), Ludwig Nobel searched for and found commercial uses for *ostaski*. Until then, other industrialists had considered *ostaski* as a worthless by-product of refining.

In order to overcome the inordinate delays incurred in shipping oil to northern markets, a pipeline was designed by Ludwig Nobel. In 1860, ships coming from the USA were able to reach Moscow and St. Petersburg in far less time than shipments from the Baku to those same northern markets. By carrying oil via the pipeline to the Caspian for shipment to the north, Baku oil was able to effectively compete with that coming from America.

During the period of unfettered capitalism, the Nobels came to dominate the petroleum industry, threatening the destruction of competitors in – and outside the petroleum industry. In 1875, the Nobels acquired the Balakhany oilfield north of Baku, presently considered to be one of the oldest of the area. As the Nobels acquired more fields, they constructed refineries using improved technology, expanded the transportation system, extended a pipeline from the fields in Baku to various outstretched refineries, and organized a distribution network. Over the objection of the competing oil refineries, the Crown even permitted the Nobels to run their own tank cars on the state-operated railroads. In short, the Nobel Brothers Naphtha Company (incorporated in 1878) had not only revolutionized but had also dominated the industry.

Realizing that shipping oil by any means other than bulk would be uneconomical, in collaboration with Sven Almquist of Sweden's Lindholmen-Motala, Ludwig designed and constructed an oil tanker. In 1878, as a part of the comprehensive expansionary program depicted above, the world's first oil tanker, named the *Zoroaster*, was put into service. The *Zoroaster* transported oil from the Baku, across the Caspian Sea, and on to Astrakhan. From Astrakhan, the oil was shipped via the Volga, then by railroad it was shipped onto the inland markets of Russia [14]. In itself, the feat was note-worth-

[14] For a more extensive discourse on the world's first oil tankers designed and constructed by Ludwig Nobel, see TOLF (1976, pp. 50-60).

y, but equally important were its consequential benefits. It proved to be a classic example of adopting import substitution and cost-reducing measures in a developing economy.

Although the *Zoroaster's* tanks were made large enough to make its use economical, the ship was also designed small enough to negotiate the tight water lanes routed. Two more tankers of identical design were added after the *Zoroaster's* proven success; these were followed by a new generation of tankers during the 1880s. The new tankers made Russia a solid competitor in the international oil markets. The United States in particular felt the challenge.

The Nobels were not alone in the pursuit of wealth in the Baku. The Nobels had refused to help finance the Tbilsi to Batum Railroad, fearing that by so doing their dominance of the Baku trade would be compromised. Their refusal resulted in the French Rothschilds entering the petroleum industry during the 1880s and consequently providing funds for the railroad. Having made investments in oil elsewhere, the Rothschilds were anxious to have the opportunity to compete in the Baku. The Germans, as well as prominent Russian nationals, such as A.J. Manteshev, were also active in the development of the Russian oil industry.

By the close of the century, over 200 companies representing several nations were actively engaged in drilling wells, refining and producing oil, and distributing oil products. Although the collective efforts of numerous investors contributed to catapulting the Russian oil industry into a position of world leadership, no other single enterprise matched the accomplishments of the Nobel Brothers Naphtha Company, Inc. By 1883, the Nobels had cornered 50 percent of the kerosene business which prompted competitors to vociferously call for a return to state intervention in the oil industry.

The Oil Regions and the Companies

As stated earlier in this paper, the Russian oil industry was centered in the Caucasus during the 1870-1900 period; the Baku area accounted for 90 percent of the national output of oil and some 45 percent of the world total. Oil production rose almost 2-1/2 times in the Baku during the last eight years of the nineteenth century. When the Baku output was complemented by the output of the Grozny fields which became commercially productive in 1893, Russia became the world leader in oil production within the decade which followed [15].

[15] The Grozny oilfields, located on the northeastern slope of the Caucasus Mountains, began

Mother Nature was not parsimonious in bestowing upon the Baku certain natural advantages which made the oil pools highly exploitable, at least during the early years. First, there were at least 20 oil horizons in the Baku area. Second, the deposits were often located very near the earth's surface. Third, the deposits were frequently clustered within a fairly small area. Fourth, the subsoil contained considerable quantities of oil, as stated earlier. Fifth, not infrequently the pools of oil were naturally pressurized.

Clearly, drilling through the soft subsoil was not difficult – even with inferior equipment. Moreover, it was unnecessary to drill deeply into the soil, and because of the concentration of the pools in a restricted geographical area, the chances of striking oil were much better than average. Also, given the abundance of oil within those subterranean reservoirs, many wells flowed steadily for some time before showing sign of drying up. Pumps and mechanical bailing equipment were not necessary because of the existing underground pressure. At the turn of the century, some forty gushers alone were producing some 1,835,000 tons of crude oil annually. In 1882 the gusher in Bibi-Eybat had provided 480,000 tons of oil in just one month [16].

An Overall View, 1870-1900

Some additional aspects of the Russian oil industry during the 1870-1900 period ought be considered. Table 2 shows the oil production of the entire world, the United States, and Russia for select years from 1870 to 1905. In addition, the annual output of the two nations in absolute and relative terms appears in the table.

A number of important observations may be made by carefully scanning the data in Table 2. Clearly, during the decade of the 1870s, the United States enjoyed world primacy in the production of oil, producing over 86 percent of the world output. The 1880s proved to be a period of transition for the United States and Russia. In 1895, the United States still held world leadership by producing slightly over 50 percent of the world output of 14.3 million metric tons. However, the Russian oil industry was rapidly gaining momentum. As the century came to a close, Russia replaced the USA as the leading world producer of crude oil. By 1899, Russia was

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production as early as 1833, but the wells were not commercially exploited until six decades later. The principal areas of production in the Baku were Balakhany, Romany, and Sabunchi to the north and Bibi-Eybat to the South. By 1901, the combined output of these areas constituted a major share of the world's oil supply.

[16] HASSMANN, (1953, p. 24).

TABLE 2

RUSSIAN OIL PRODUCTION, COMPARED TO THE U.S.A. AND THE WORLD, FOR SELECT YEARS, 1870-1905

(Expressed in Millions of Metric Tons)

Year	World	Russia		U.S.A	
		Output	Percentage	Output	Percentage
1870	.798	.028	3.51	.711	89.10
1875	1.373	.096	6.99	1.187	86.45
1880	4.131	.413	10.00	3.552	85.98
1885	5.060	1.916	37.87	2.954	58.38
1890	10.547	3.949	37.44	6.192	58.71
1895	14.271	6.350	44.50	7.147	50.08
1898	17.201	8.479	50.86	7.481	43.49
1899	18.049	9.077	50.29	7.711	42.72
1900	20.525	10.429	50.81	8.596	41.88
1901	23.044	11.721	50.86	9.376	40.68
1902	25.022	11.085	44.30	11.994	47.93
1903	26.821	10.403	38.79	13.574	50.61
1904	29.996	10.809	36.07	15.820	52.74
1950	29.602	7.564	25.55	18.203	61.49

Source: The basic data was drawn from HASSMANN, Press, (1953, pp. 147-48).

producing 50.9 percent of the world's crude oil. The data in the table also indicate that Russian superiority continued until 1903, when the leadership reverted to the United States. Not until some 70 years later (1974) was the USSR to regain world leadership, when the Soviet Union produced 458.95 million metric tons to the USA's 432.8.

Conclusion

During the closing decades of the nineteenth century, the Russian petroleum industry burst forward. The rapid growth and development occurred not necessarily because of any special incentive policies created by government, but in spite of certain impeding government restrictions. Only after the Kokorev and Mendelev inquiries did unbridled capitalism dominate the oil industry, and then for but a decade. A number of factors contributed to the expansion of the oil industry, especially in the Baku region which produced about 90 percent of Russian oil during the 1870-1913 period.

The favorable geological features of the "First Baku", a generous inflow of foreign capital, growing domestic markets for kerosene and other petroleum products, and the expansion of Russia's infrastructure all contributed to the industry's growth. By underscoring Russia's relative economic backwardness, the bitter defeat at Sevastopol in 1855 sparked a new wave of westernization. Moreover, the willingness of foreign entrepreneurs to provide their talent and expertise coupled with the industrial plans of Finance Ministers Vyshnegradskii, Witte, and von Plehve added impetus to the oil industry. However, in some cases, such as the Nobels, preferential treatment also made a difference among entrepreneurs. The Nobel success story is, in a measure, ascribable to such treatment, for they were (1) allowed the advantage of a pipeline to ship oil, (2) given special permission to ship kerosene across the Caspian using their own tankers, and (3) privileged to place their own tank cars on the state rail system.

The dawn of the new century found Russia in the forefront as a world oil producer, but its superiority was shortlived. Although signs of depletion had become apparent at some wells, the violent industrial, political, and social strife seizing the nation before and at the turn of the century paralyzed various industrial centers, including the Baku. Other deleterious factors were the water problem at the wells caused by stoppages, the imposition of new higher rail tariffs, and with some exceptions, the continuous use of obsolete methods in drilling and extraction. By 1905, Russia had clearly lost its position of world primacy in oil production; its oil output had shrunk to 25.5 percent of the world's. Among the world's oil producers, Russia's relative status reverted to that of some two decades earlier, and not until 1974 was Russia to once again claim primacy as the world producer of oil.

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LO SVILUPPO DELL'INDUSTRIA PETROLIFERA RUSSA NEL XIX SECOLO

Nel 1974 l'Unione Sovietica raggiunse il primato mondiale come produttrice di petrolio grezzo con i suoi 459 milioni di tonnellate superando gli Stati Uniti (433 milioni). Questo successo sovietico è in gran parte attribuibile alle basi della sua industria petrolifera che furono poste nel periodo 1870-1900.

L'autore di questo articolo passa in rassegna alcune delle politiche che hanno modellato l'industria petrolifera russa nella regione del Baku. L'attenzione è qui volta anche a parecchi imprenditori stranieri, in particolar modo i fratelli Nobel, che sono considerati fra i principali imprenditori del 19° secolo cui va il merito di avere posto le prime basi dell'industria petrolifera sovietica.

Inoltre, l'autore si concentra su alcune delle cause che hanno contribuito alla crescita di questa industria man mano che la nazione si avvicinava al volgere del secolo.

RECENSIONI

BRENNAN G. - J.M. BUCHANAN: *The Reason of Rules - Constitutional Political Economy.* 1985, Cambridge, Cambridge University Press, pp. 153, Lst. 22.50, US $ 34.50.

L'obiettivo degli autori è quello di comprendere il funzionamento delle istituzioni politiche alternative come premessa per una più idonea scelta tra regole: in questa prospettiva quella che in altre opere era definita "political economy" diventa "constitutional political economy". Gli autori osservano che le attenzioni degli economisti nel nostro secolo si sono concentrate sui *fallimenti del mercato* e si sono posti come obiettivo quello di "fornire consigli" su come evitare tali fallimenti. È sorprendente come questo approccio non sia stato esteso ad istituzioni diverse dal mercato: le scelte politiche e quindi i fallimenti politici sono rimasti al di fuori dell'attività degli economisti. Da qui l'implicita ipotesi che i gruppi politici agiscano come "moral super persons" in grado di massimizzare la funzione del benessere sociale. Per Brennan e Buchanan gli individui sono moralmente uguali; ne discende che il costituzionalismo è necessariamente "contrattualismo".

Il volume si divide in quattro parti. I primi quattro capitoli si occupano di problemi generali analizzando particolari tipi di regole. I capitoli 5 e 6 si occupano della dimensione temporale delle scelte private, distinte dal contesto delle scelte sociali o collettive. Ne risulta che l'individuo sconta con maggior difficoltà in termini razionali in un contesto di scelte collettive, da qui la ragione, tra l'altro, per la fissazione di regole. I capitoli 7 e 8 si occupano del rapporto tra regole e giustizia.

Il capitolo 9, che è forse il più importante del volume, si pone l'interrogativo se in democrazia siano possibili cambiamenti costituzionali e *prima facie* risponde negativamente. Ciò non significa che come studiosi e come cittadini non possiamo modificare le regole del gioco che giochiamo. Le regole sono costruzioni umane. Gli individui compiono tre tipi di scelta: in primo luogo hanno l'alternativa regola-non regola che è concettualmente diversa dall'alternativa tra diverse regole. A sua volta la scelta tra regole alternative differisce dalle opzioni all'interno di regole esistenti. La prima alternativa può essere espressa anche in termini di anarchia-ordine; su tale argomento è andata crescendo una letteratura di notevole interesse metodologico all'interno della Public Choice a partire dagli anni '70. La rilevanza analitica della seconda alternativa è pure stata trattata da Buchanan in quello che mi sembra il suo contributo fondamentale in questa direzione (Vedi *The Limits of Liberty*). Tuttavia, questo lavoro non è ripetitivo come apparirà chiaramente a chi vorrà leggere il capitolo 9 con l'attenzione che merita.

Poiché – è questa la sostanza del volume – non necessariamente emergono le regole migliori, cioè le più efficienti (tesi questa sostenuta da Hayek), spetta ai governi sostituire le vecchie regole con nuove regole tenendo conto che scelte a livello costituzionale richiedono tempi lunghi poiché cambiamenti troppo frequenti sono incompatibili con il concetto stesso di regola che implica stabilità in quanto deve fornire informazioni per rendere i comportamenti prevedibili. Occorre segnalare che la posizione costruttivistica di Brennan e Buchanan fa emergere come le costituzioni siano fenomeni prodotti dalle azioni umane e come tali "artifactual" e non naturali. In nessun modo tale costruttivismo suggerisce il legame con la massimizzazione del benessere sociale in quanto ciò che emerge dagli scambi è un "outcome" che può esser noto solo *ex post*.

GIUSEPPE EUSEPI

ØKSENDAL B.: *Stochastic Differential Equations*. 1985, Berlin, Springer-Verlag, pp. 205, D.M. 42 (softcover).

Le equazioni differenziali stocastiche – oltre al loro interesse matematico – presentano un ampio spettro di applicazioni in diversi campi, come, ad esempio, nella teoria stocastica del potenziale, nel problema dell'arresto ottimo (optimal stopping), nella selezione del portafoglio ottimo.

Il volume origina da un corso tenuto all'Università di Edimburgo nel 1982 ed è caratterizzato da una esposizione semplice e lineare, in contrapposto alla letteratura corrente su questo argomento che, in generale, tende a sottolineare il rigore e la validità delle dimostrazioni.

Nell'introduzione vengono considerati, da un punto di vista elementare, alcuni problemi (sviluppo di una popolazione, problemi di filtraggio, arresto ottimo, controllo stocastico ecc.) nei quali le equazioni differenziali stocastiche giocano un ruolo essenziale.

Nel capitolo successivo vengono sviluppati i concetti matematici essenziali alla comprensione del seguito come quello di variabile casuale, di processo stocastico e di movimento di Brown.

Queste nozioni sono la premessa per il capitolo successivo dedicato all'ormai famoso integrale di Ito, alla sua algebra nonché al confronto con l'integrale di Stratonovich.

I due capitoli successivi sviluppano, rispettivamente, il calcolo stocastico (con la formula di Ito) e l'impiego di questo calcolo per risolvere alcune equazioni differenziali stocastiche.

Seguono un capitolo destinato alla soluzione del problema del filtraggio lineare usando il calcolo stocastico e un altro, alquanto vasto, dedicato alla diffusione nel quale si considerano diversi aspetti di questo problema, dalla diffusione di Ito, alle variazioni casuali riferite al tempo, al moto di Brown su una sfera unitaria.

Alcune applicazioni delle equazioni differenziali stocastiche sono studiate nei tre capitoli successivi dedicati, rispettivamente, alle equazioni differenziali parziali, alle regole dell'arresto ottimo e al controllo stocastico.

Due appendici, una ampia bibliografia e la lista dei simboli chiudono il libro.

Il volume è destinato ai lettori non introdotti in questo argomento e che desiderano applicare i metodi esposti ai propri campi di ricerca. Non manca un numero piuttosto elevato di esempi scelti in campi diversi.

Dai brevi cenni sopra riportati si rileva subito che il libro richiede al lettore una solida preparazione matematica con la conoscenza di aspetti particolari.

LUIGI VAJANI

RUYMGAART P.A. and SOONG T.T.: *Mathematics of Kalman-Bucy Filtering*. 1985, Berlin, Springer-Verlag, pp. 170, D.M. 84 (hard cover).

La data d'introduzione delle tecniche di filtraggio sviluppate prima da Kalman (per il caso discreto) e successivamente da Kalman-Bucy (per il caso continuo) risale alla metà del 1950. In questi ultimi trent'anni circa, questo metodo ha trovato applicazioni in quasi tutte le aree delle scienze applicate. Sorto originariamente per le applicazioni aerospaziali si è ora esteso ad altre aree ingegneristiche ed anche alle scienze sociali e a quelle biologiche e mediche.

Questo volume tratta prevalentemente il caso continuo del filtro di Kalman-Bucy ed è basato sull'utilizzazione dello spazio di Hilbert. I concetti fondamentali relativi a questo spazio sono sviluppati nel primo capitolo il quale comprende anche una succinta esposizione dei principi fondamentali della teoria delle probabilità.

Notevole sviluppo viene dato al calcolo « in media quadratica » (convergenza in media quadratica, differenziabilità e integrabilità in media quadratica, ecc.) il quale è sviluppato nel secondo capitolo, assieme alla teoria del processo di Wiener-Levy e all'equazione di Riccati.

L'ultimo capitolo introduce un teorema dovuto a Liptser e Shiryayev il quale consente di chiarire la dipendenza fra lo stimatore di Kalman-Bucy e i disturbi legati ai processi dinamici.

Chiudono il volume le soluzioni di alcuni esercizi proposti nel testo ed una lista di opere da consultare per chi volesse approfondire alcuni specifici argomenti.

Il libro è particolarmente indicato per coloro che volessero avere una sicura guida nello studio di questo utilissimo strumento d'indagine.

LUIGI VAJANI

DACUNHA-CASTELLE D. and DUFLO M.: *Probability and Statistics*. 1986, Berlin-New York, Springer-Verlag, Vol. I, pp. 362, D.M. 89. Vol. II, pp. 410, DM. 86 (hard cover).

L'opera è di due autori dell'Université de Paris-Sud e Nord.

Il volume I si sviluppa in otto capitoli il primo dei quali, dal titolo « census » affronta la descrizione del censimento per passare alle prime definizioni di probabilità nel discreto. Segue un capitolo che studia le prove disotome e introduce il concetto di intervallo di confidenza, con interessanti applicazioni al controllo statistico di qualità. I capitoli successivi sono orientati prevalentemente verso gli aspetti matematici dei vari problemi.

Il capitolo terzo esamina i modelli probabilistici alla luce della teoria della misura, con particolare riferimento alla teoria dell'integrazione e alla distribuzione di una funzione misurabile. I capitoli quarto, quinto e sesto esaminano, prevalentemente, questioni più strettamente statistiche come la stima di una distribuzione, i test non parametrici, i campioni di tipo gaussiano, il test « chi quadrato », la teoria della regressione, le catene di Markov, ed altre questioni ancora.

Il capitolo settimo inizia con un'esposizione approfondita delle distribuzioni binomiale e di quella gaussiana, completando i concetti prima delineati relativi a queste due distribuzioni così fondamentali in statistica. Prosegue, questo capitolo, mostrando come si possano misurare le informazioni fornite da esperimenti statistici sui parametri da cui dipendono. Affronta, infine, il concetto di massima verosimiglianza mettendo in evidenza come esso possa permettere di ottenere certi stimatori e certi test aventi particolari proprietà.

Il capitolo che segue è dedicato alle decisioni su basi statistiche e studia le statistiche bayesiane, le ottimali proprietà di certi rapporti di verosimiglianza nonché il principio di invarianza.

Il volume è sviluppato a livello avanzato, con l'impiego di diverse matematiche piuttosto elaborate.

Al primo volume, dedicato specificatamente al Calcolo delle Probabilità, fa seguito il secondo che tratta, in modo particolare, dei processi stocastici. Esso inizia con un capitolo introduttivo (capitolo zero) in cui sono esposti i primi concetti sui processi casuali. Vengono esaminati gli eventi casuali che si sviluppano nel tempo, i fondamenti della teoria della misura nonché il concetto di convergenza nelle distribuzioni.

Alle serie temporali è dedicato il primo capitolo il quale tratta dei processi stocastici di secondo ordine, i processi spaziali con incrementi ortogonali e le statistiche necessarie per studiare le serie temporali come l'autocovarianza e la densità spettrale.

Il secondo capitolo tratta delle martingale nel tempo discreto le quali costituiscono la base, sia per i capitoli successivi, sia per l'estensione al caso continuo. È questo uno dei più ampi che tratta, fra l'altro, del problema dell'arresto di un processo (*stopping problem*), della convergenza, delle sub-martingale e di alcuni specifici problemi di massima verosimiglianza, nonché dell'informazione di Kullback.

Le statistiche asintotiche costituiscono l'oggetto del successivo capitolo, nel quale sono illustrati i metodi in relazione, prevalentemente, al caso di campioni indipendenti, di serie temporali e di processi di Poisson. Vengono esaminati, per citare solo alcuni aspetti, la stima a massima verosimiglianza in un campione, il test basato sul rapporto di verosimiglianza e il test « chi quadrato ».

Il capitolo quarto considera le catene di Markov ed esamina, in particolare, il problema generale degli stati ricorrenti e di quelli transitori, per terminare con l'esame delle statistiche che si incontrano nello studio di queste catene.

Il quinto capitolo dal titolo « Decisioni passo a passo » è dedicato alla regole di fermata ottima (*optimal stopping*), al controllo delle catene di Markov, alle statistiche relative ai processi sequenziali e ai test di verosimiglianza.

I due capitoli che seguono sono relativi il primo ai processi numeralizzabili (*counting processes*) e precisamente ai processi di rinnovo, alle passaggiate a caso, al processo di Poisson, mentre il secondo è orientato verso i processi continui nel tempo, come le martingale nel continuo ed i processi con traiettorie continue.

L'ultimo capitolo è dedicato agli integrali stocastici che in questi ultimi anni hanno trovato vastissime applicazioni. Vengono studiati la formula di Ito, il calcolo stocastico, i movimenti di Brown, la regressione e la diffusione nel continuo e il processo di Ornstein-Uhlenbeck.

Un'ampia e aggiornata bibliografia, relativa prevalentemente a trattati ed articoli ad alto livello completa il volume. Va inoltre notato che alla fine di ciascun capitolo sono inserite delle note bibliografiche le quali forniscono alcune indicazioni sui più recenti studi relativi al capitolo stesso.

L'opera, estremamente interessante per la vastità degli argomenti trattati e per il rigore dell'esposizione, richiede una solida preparazione sia matematica che statistica.

Indubbiamente l'insieme dei due volumi fornisce una visione molto ampia del metodo statistico, con l'esame anche di problemi particolari che possono interessare specifiche discipline.

LUIGI VAJANI

LIBRI RICEVUTI

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RISERVE E FONDI VARI L. 410.003.791.910

Il 28 aprile 1987, si è svolta l'assemblea ordinaria degli Azionisti.
L'assemblea ha approvato il bilancio al 31 dicembre 1986, la cui regolarità è stata certificata dalla Peat, Marwick, Mitchell & Co. Gli impieghi complessivi sono saliti a L. 3.437 miliardi. L'utile di esercizio, dopo ammortamenti e accantonamenti per complessive L. 63,9 miliardi, è risultato di L. 40 miliardi che sono stati destinati per L. 19,3 miliardi a riserva ordinaria dell'Istituto e della Sezione OO.PP., per L. 14,9 miliardi a riserva straordinaria dell'Istituto, e per L. 5,8 miliardi agli azionisti, con un dividendo di L. 480 ad azione.

SINTESI DEL BILANCIO AL 31 DICEMBRE 1986

ATTIVO	(in milioni)
Disponibilità presso Istituti di credito e in cassa	L. 183.467
Titoli di proprietà	L. 231.602
Finanziamenti	L. 3.437.459
Rate maturate	L. 434.645
Altre attività	L. 299.671
Partecipazioni, immobili, mobili, impianti e macchinari	L. 103.222
Totale	L. 4.690.066

PASSIVO	(in milioni)
Titoli in circolazione	L. 3.796.348
Altre passività	L. 319.376
Fondi rischi su crediti	L. 146.331
Altri fondi	L. 59.904
Capitale sociale	L. 96.000
Riserva ordinaria	L. 206.208
Riserva straordinaria	L. 22.278
Riserva per acquisto azioni sociali	L. 300
Azionisti per versamenti in conto aumento capitale sociale	L. 3.288
Utile netto esercizio 1986	L. 40.033
Totale	L. 4.690.066

Il dividendo è esigibile dal 18 maggio p.v. presso la sede sociale, ovvero presso la Cassa di Sovvenzioni e Risparmio fra il Personale della Banca d'Italia nelle sedi della Banca stessa in Ancona, Bari, Bologna, Cagliari, Firenze, Genova, Livorno, Milano, Napoli, Palermo, Roma, Torino, Trieste e Venezia, nonché in Milano presso gli uffici delle Direzioni Generali dell'Istituto Centrale di Banche e Banchieri e dell'Istituto Centrale delle Banche Popolari Italiane.

Il Consiglio di Amministrazione è così composto: Presidente: dr. Antonio Occhiuto; Vice Presidenti: prof. Tancredi Bianchi e dott. Corrado Danieli; Consiglieri: dott. Giovanni Auletta Armenise, dott. Maurizio Bonsi, dott. Lamberto Cantuti Castelvetri, dott. Franco de Majo (Segretario del Consiglio), dott. Mario Fornari, ing. Ettore Lolli, dott. Giuliano Monterastelli, ing. Carlo Odorisio, prof. Guido Mario Rey, dott. Carlo Rivano, prof. Piero Schlesinger, dott. Ivo Turchetti.
Il Comitato Esecutivo è così composto: dott. Antonino Occhiuto, prof. Tancredi Bianchi, dott. Corrado Danieli, dott. Franco de Majo, dott. Carlo Rivano, dott. Mario Fornari, ing. Carlo Odorisio; Segretario del Comitato: il Direttore Generale; Avv. Alberto Citoni.
Il Collegio Sindacale è così composto: Presidente: dott. Fulvio Innorta; Sindaci effettivi: geom. Luigi Bacci, rag. Ottavio Fontanesi, avv. Giorgio Luciani, dott. Vladimiro Pirovine.

Eni

LA STRUTTURA ORGANIZZATIVA DEL GRUPPO

La struttura organizzativa del gruppo ENI è basata su una holding, l'ENI, che detiene attualmente la totalità, o la quasi totalità, del capitale di 13 principali Società Caposettore, alle quali fanno capo un insieme di oltre 300 Società, delle quali oltre un terzo ubicate all'estero. Le Società operative sono soggette alla stessa regolamentazione ed hanno la stessa natura di impresa prevista, sia in Italia sia all'estero, per le Società per Azioni il cui capitale sia nelle mani di privati Azionisti. Tale struttura organizzativa consente un elevato grado di flessibilità operativa e gestionale, ed è in grado di evolversi, in modo autonomo, secondo le esigenze poste dalla situazione economica ed industriale dei vari settori e mercati nei quali l'ENI si trova ad operare. Nei confronti delle Società operative l'ENI svolge funzioni di direzione e coordinamento nelle attività di programmazione e controllo, nelle attività all'estero, nelle politiche del personale e dei rapporti con la realtà esterna.

Particolarmente significative sono le funzioni dell'ENI in campo finanziario; l'ENI, infatti, oltre a svolgere una funzione di coordinamento nei rapporti con gli operatori finanziari nazionali ed internazionali, propone le politiche ed i piani di copertura finanziaria del Gruppo, sovraintende alla loro attuazione e controllo, e pianifica e coordina le operazioni finanziarie di Gruppo. Tali funzioni vengono esercitate, oltre che in accordo con le strutture delle Società Caposettore, attraverso una rete di Società finanziarie ubicate sia in Italia sia all'estero.

Eni

Ente Nazionale Idrocarburi
Sede in Roma
Uffici:
00144 Roma, piazzale Enrico Mattei, 1
telex: 610082 - 610086
telegrafo: Enidro-Roma

Agip

Esplorazione e produzione di idrocarburi; approvvigionamento di greggi; ciclo del combustibile nucleare; sviluppo e impiego di fonti rinnovabili di energia (geotermia e fotovoltaico); attività nel settore dei minerali non ferrosi.

AgipPetroli

Raffinazione e distribuzione di prodotti petroliferi. Fornitura di servizi per il risparmio dell'energia, per la razionalizzazione dei consumi e l'uso di fonti energetiche diverse dal petrolio.

Snam

Approvvigionamento, trasporto, distribuzione e vendita di gas naturale.
Trasporto di idrocarburi liquidi.

AgipCarbone

Ciclo integrato del carbone: ricerca e coltivazione mineraria, logistica e trasporto, trasformazione, commercializzazione su scala internazionale, ricerca scientifica e tecnologica per sviluppare e diversificare l'utilizzo del carbone e dei derivati.

Nuova Samim

Produzione, trasformazione e commercializzazione di metalli non ferrosi da minerali e da rottami e residui civili ed industriali; estrazione e lavorazione dei marmi, produzione di acido solforico e derivati del bario.
Produzione e trasformazione di materiali abrasivi e ceramici avanzati.

EniChem

Petrolchimica di base, materie plastiche, gomma sintetica, prodotti chimici per l'agricoltura, fibre sintetiche, materie prime per detergenti, tecnopolimeri, chimica fine, prodotti farmaceutici.

Saipem

Perforazione terra e mare.
Costruzione terra (pipelines, impianti industriali, ecc.).
Lavori mare (sealines, piattaforme, terminali, ecc.).

Snamprogetti

Studio, progettazione e realizzazione di impianti chimici e petrolchimici, di raffinazione, di trattamento gas, condotte in terra e in mare, tecnologia offshore, impianti industriali, impianti per l'ecologia e grandi infrastrutture.

NuovoPignone

Progettazione e costruzione di macchine, apparecchiature e strumenti per l'industria degli idrocarburi, petrolchimica, elettrica, nucleare e tessile.
Sistemi modularizzati e sistemi di automazione.

Savio

Produzione e fornitura di macchine per l'industria tessile.

Lanerossi

Industria tessile e dell'abbigliamento.

Sofid

Finanziamento di attività industriali e commerciali del gruppo ENI.

Eni Int. Ho

Compravendita e gestione di partecipazioni e titoli; finanziamento delle attività del gruppo ENI all'estero.

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